IMPROVE YOUR THINKING

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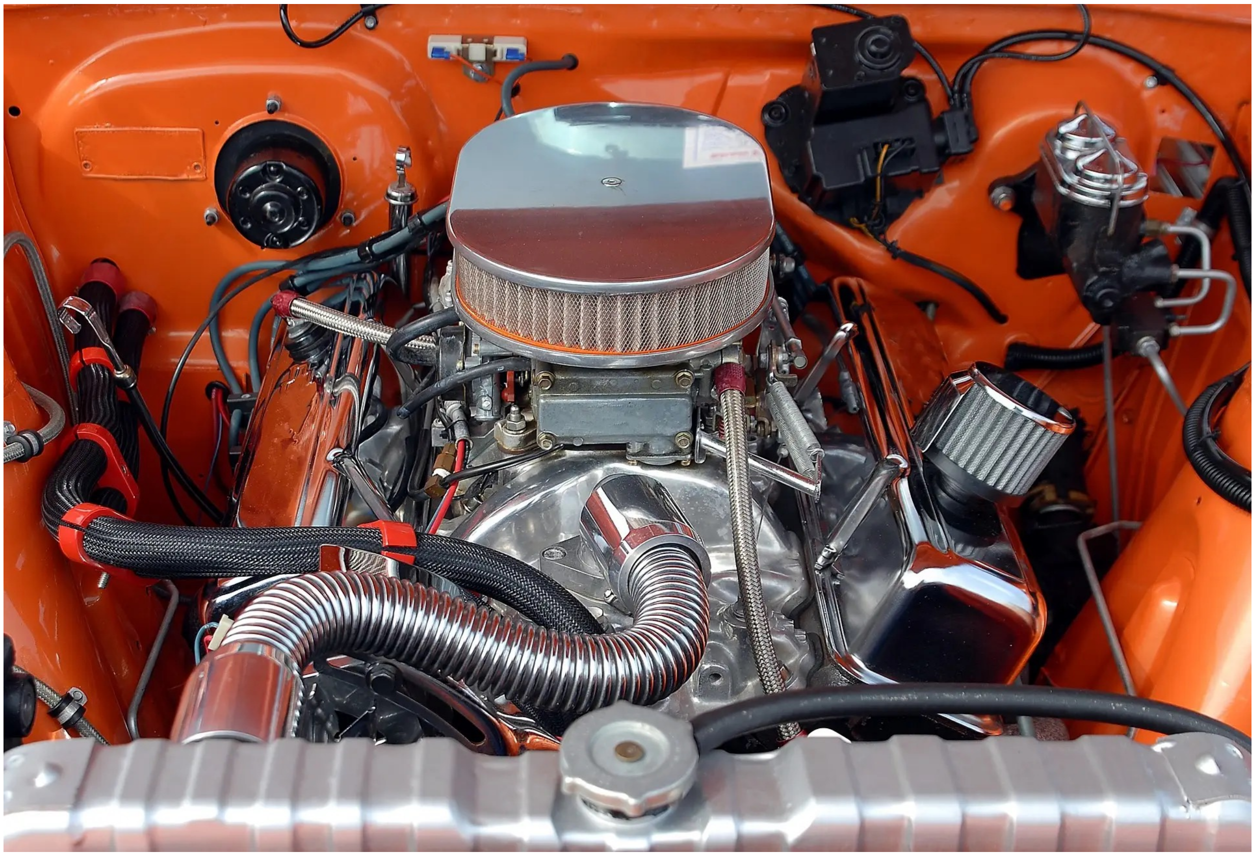
WHY STUDY LOGIC?

This is a very common and quite understandable question especially given the difficulty of the subject and the abstract nature of much of formal logic. What possible purpose could be served in learning logic? When will you ever use logic? Let's look at some reasons and attempt to make the case for the importance of learning logic.
I think several distinct arguments can be made.

1. The relevance argument:
A common question concerning logic is how it could be relevant to the major of the student or their career. This is a stunning point if one thinks about it in light of the nature of logic which we'll address below. However, one consideration is offered by the economist Thomas Sowell. Relevance is something you can only assess after you've learned a subject. You can't tell until then whether something is relevant to your life or not.

Closely related to this point is the fact that none of us knows for sure what will happen next in our life and so we can never be absolutely sure that a subject, any subject, will not be relevant. Think about it. As a 20-year-old you might say, I know I'm never going to use this in my career or life. Assuming (which is a safe assumption) that you'll live for 50–70 more years, how can you say this? How do you know for that length of time what will be relevant to your life and what won't? I took logic as a sophomore majoring in telecommunications and could have easily believed at the time that I might never use logic again. Who knew I'd end up teaching it!?

2. The nature of logic argument:
Logic is concerned with training the mind to think clearly. Given this, let's ask the question again: When will I ever need clear thinking? Now, the importance of learning logic should be crystal clear. There isn't a single area of life where clear thinking wouldn't be beneficial to some degree. The real issue here is not whether logic is useful, but how can logic and what we do in a formal logic class help improve your thinking. The real question is not whether clear thinking is needed but how can learning about categorical syllogisms, truth tables, and natural deduction improve our ability to think clearly. I think I have some answers to those questions.



3. The deeper understanding argument:
One of the reasons to question the benefit of studying logic in the first place is the apparent strangeness of formal logic. It looks so different from our ordinary language use because it is a formal symbolic system abstracted from ordinary language. What we are attempting to analyze in logic is the underlying nature of inferential thinking and to do so we must inspect the form of our reasoning by separating it from its content.

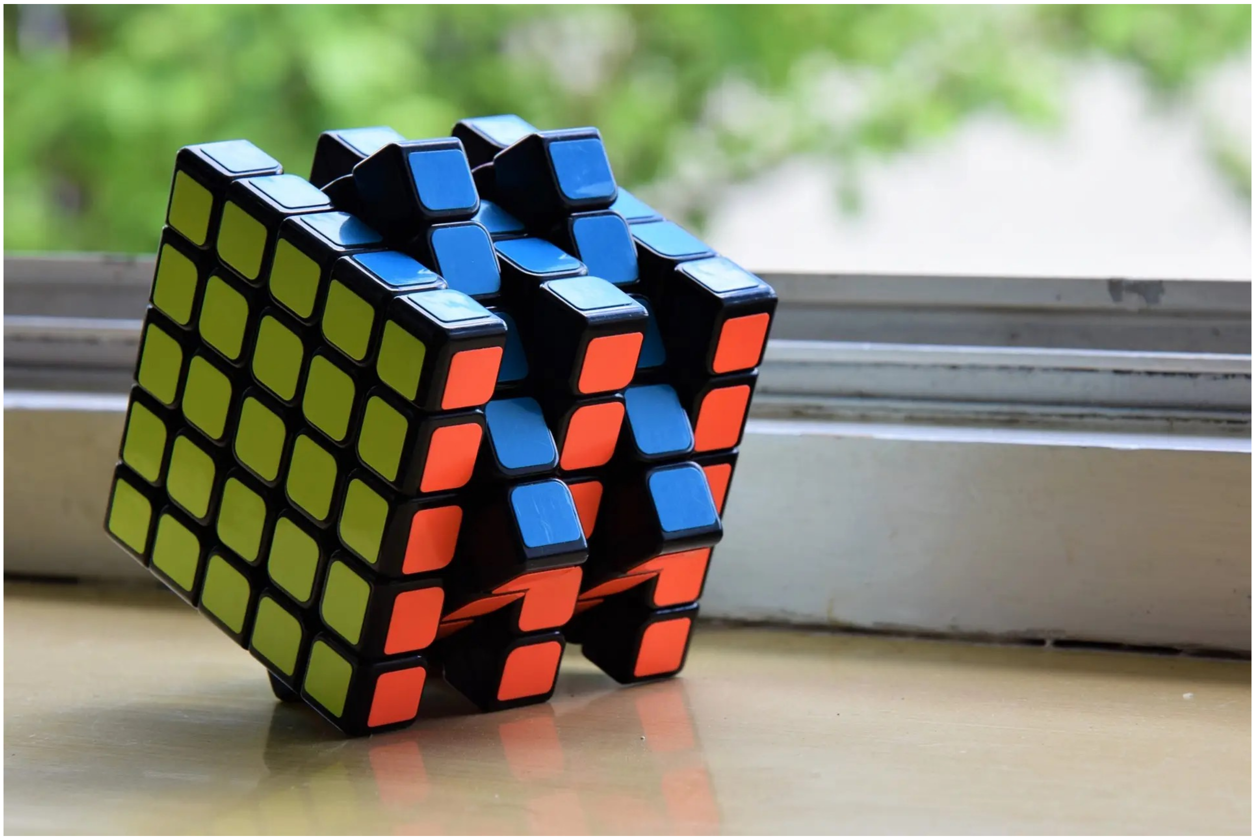
Doing so makes it look irrelevant because looking at a car engine out of context looks irrelevant to the working of the car. Think about it. If you looked at the engine of a car without ever addressing the purpose or context you would never connect that mechanism with driving your car. If you never bothered to look under the hood you might not even know there is such a thing as an engine! For all you know there's nothing under the hood. So, if you look at the engine for the first time it looks strange.

It's the same with the inner workings of thinking and reasoning. It's only after studying formal logic that you begin to see that the principles of logic are connected with ordinary thinking. So by studying the underlying depth of the subject you can get a greater appreciation of the ordinary application and in the process become better at that application.

Let's look at what formal logic (or symbolic logic) is forcing us to do, not from the standpoint of using the principles of formal logic directly, but from the standpoint of the underlying skills these principles are drawing on.

In categorical logic, you have to read statements carefully to make distinctions between terms that on the surface sound similar. You have to recognize general rules by looking at specific cases. In propositional logic, you have to recognize the general rules underlying the use of certain words and recognize these principles in different situations. Finally, in natural deduction, you have to take a set of rules and apply it in an orderly method to solve a problem.

Look at the skills being used here: rule recognition, abstraction, planning, problem-solving, making distinctions. It is these skills that logic is training you to improve and it is these skills that represent the real practical benefits of learning logic.



4. The mental exercise argument

Perhaps an analogy will help. Some people go to the gym to work out. They lift weights, do stair climbing, tread, etc. Do they do these things to improve their ability to lift weights, climb stairs, and walk? No, they do them for some other benefit which these exercises indirectly lead to. It's the same with the mind. We need some form of exercise for the mind and that's what logic is. The benefits of logic are indirect. That is, what we do in logic improves our ability to do something else.

So, why don't we just practice the thing itself instead of practicing the skills indirectly by learning logic? Well, the answer is that we do this as well but logic represents a more rigorous form of exercise. Look at it like this. When you walk to your car or to class you are getting some exercise benefit. But, you may also go to the gym. When you carry groceries from the store to your car you are getting some exercise benefits. But, you may also lift weights. Why?

Because the more casual form of the exercise isn't really vigorous enough to get the true benefit of doing the exercise. It's the same with logic. When you read a book or magazine you are getting some benefit from your mental exercise. But, you need to train your mind in a more rigorous fashion forcing it to do more involved thinking. Like all exercises, your muscles are sore at first until you get used to the exertion. In time you find you are better able to handle the exercise and your general thinking skills improve as a result.

5. The general knowledge argument

It's funny how learning works. The more you know, the more you can learn and the more you know the easier it is to learn something new. Learning, then, is about making connections. The more general knowledge you have the easier it is to make connections. So, in this vein, logic adds one more subject of knowledge to your mind allowing you to make more connections thus making the learning of anything else easier. Not only that, since the subject matter of logic is based on inferential reasoning the very skill of making connections is being learned as you go through your training in logic.

If you know something about psychology, understanding philosophy is easier because they're connected. That's an easy one to see. But there are so many ways that one field of knowledge is connected to another that we don't see immediately. Learning about logic makes it easier to learn about computers. How so? Computer programs are nothing more than logic commands. OK, but what if you're not programming a computer, only using it?

Still, there is a benefit to understanding the method of following rules, thinking in an orderly fashion, and recognizing that one step follows another. All skills that logic teaches. There are other connections to consider as well but too numerous to list here.

Given that you don't know what might be useful later in life, the obvious utility of logic in training the mind, and the connections between different subjects that make learning easier, it only makes sense to obtain as much knowledge as you can while you can get it. There's no better time than right now to add to your knowledge. Even knowledge which has no direct benefits now might be useful to you later on in a direct way and is certainly useful to you now in indirect ways.

Some students come to a logic class (and perhaps many others) with the attitude of just wanting to get the grade. That's unfortunate. First, you're paying for something you're not taking delivery on which doesn't make a lot of sense. Like going to the store and purchasing a new big-screen plasma TV and paying for it but not picking it up! Students have told me that once they leave the class they'll forget everything they've learned. That's unfortunate too. Again, you're paying for something and not taking delivery.

But also you're missing out on something that WILL benefit you in the near future and MAY benefit you later in life as well. Are you really willing to discard useful information so quickly? Think twice before doing that. Not only with logic but with every other subject you encounter. Besides, it costs you nothing to hang on to knowledge. The brain is not a sieve leaking out old information to make room for new or a small container, which must be cleared out to make room for new information. The brain is a complex organ capable of making connections between different knowledge sets, and the more connections you make the more knowledgeable you become. Imagine how much better at information processing and using you'd be if you learned the basics of making connections, inferences, arguments, and deductions. Logic teaches all of those skills!

In academia, we've known for years that people with limited knowledge are easy targets for scam artists, tricksters, politicians, and demagogues. What you need to know is that the scam artists, tricksters, politicians, and demagogues know this too. Francis Bacon was right; knowledge is power. And protection. You spend money to protect your computer against viruses, your cars against theft, your health against illness. You're also spending money to protect your mind against harm. The product you purchase to do this is NOT a grade. It is NOT a piece of paper called a degree. It's the knowledge behind them that will help you. You paid for it. Take it!

UNIT ONE: THE BASICS

THE S.E.A.R.C.H FORMULA

This formula comes from a book titled How to Think About Weird Things. It is a great guide to assessing any claim and possible explanations for it. SEARCH is an acronym that stands for:

State the claim.
Examine the Evidence for the claim.
Consider **A**lternative hypotheses.
Rate, according to the **C**riteria of adequacy, each **H**ypothesis.

As we'll see throughout the semester there are several parts of this process that are especially difficult. Among these, the most difficult for people seems to be considering alternative hypotheses. Another way of putting this is to consider the evidence against the claim you are evaluating. The reason this is so difficult is due to something called confirmation bias: the tendency we all have to look only for evidence that confirms our own beliefs. But, to really evaluate any claim, you need to look at all the evidence, not just the evidence in favor of the claim.

So, by all means, look at all the evidence for the claim you are investigating. But, then do what may seem counterintuitive, even counterproductive: look at the evidence against your claim. Be sure to look at it with an eye toward fair evaluation. Don't simply look at it to dismiss it. Above all, don't look at a biased form of this evidence. You know what I mean. Don't go to a source that is arguing for the position you agree with and simply take what they say about the evidence against the claim. Look to the best possible sources of evidence against your claim. That way you know you are really evaluating your claim not simply endorsing what you already agree with.

To ensure you are really doing this rate each hypothesis you are considering according to the criteria of adequacy. These criteria are

Testability: If you cannot even figure out how to go about determining if your theory explains the evidence, you don't have a good theory. To be testable means your hypothesis "predicts something more than what is predicted by the background theory alone." In short, we need this criterion because if there's no way to tell whether a theory is true or false it's really no good to us.

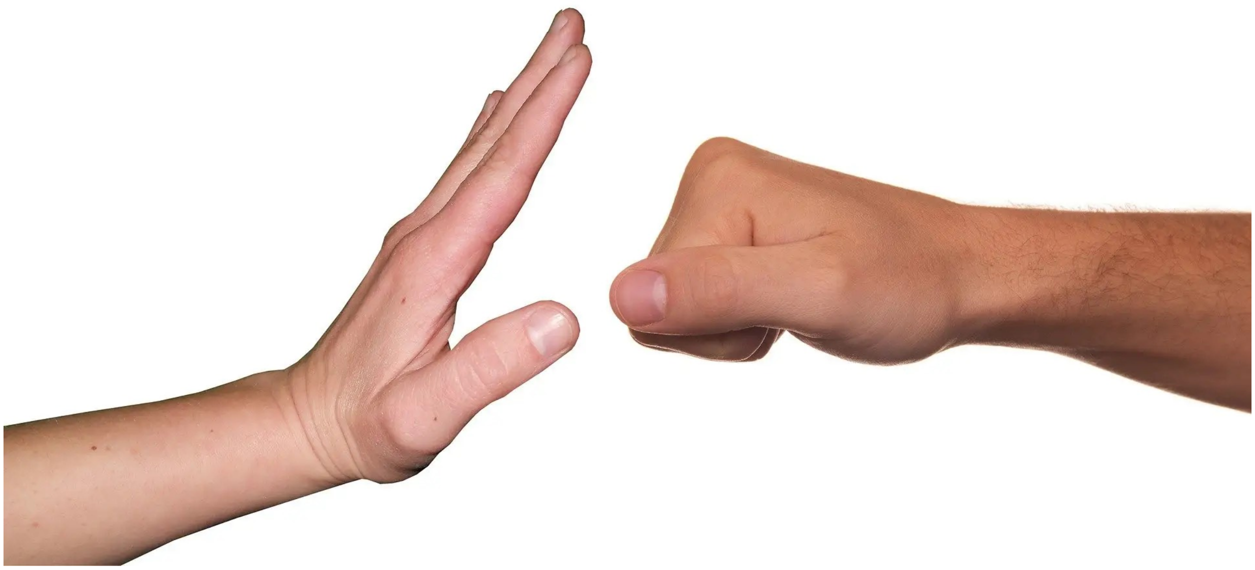
Fruitfulness: What this means is that a good theory should make novel predictions. It should not only account for the evidence at hand but be able to address evidence that comes in later and even predict such new evidence.

Scope: That is, a good theory explains a wide field of evidence. One of the differences between theories and hypotheses is their scope. Hypotheses address specific questions whereas theories attempt to provide a broad explanatory device. Theories that can explain a wide array of things are preferred, other things being equal, to more narrow theories.

Simple: This term should not be confused with simplistic. Many scientific theories are complex in terms of our ability to understand them but simple in the sense that they postulate fewer underlying entities or assumptions.

Conservative: Not in the political sense of the word. Rather, it should fit in with other things we know. If we have an explanation for something that we think is fairly certain and accurate then a new theory should fit in with that prior explanation. If it doesn't fit that may indicate our prior knowledge is flawed. We have to be open to that possibility but the burden of proof is on the new theory.

ARGUMENTS, PREMISES, AND CONCLUSIONS



Logic is defined as the science of evaluating arguments. However, before we can evaluate them we must be able to do some identifying. First, we will focus on identifying the parts of arguments: premises and conclusion. Secondly, in the next lesson, we will focus on identifying and distinguishing arguments from non-arguments. Thirdly, in the lesson following that one, we will focus on identifying different types of arguments. Once we can make these identifications and distinctions we will be prepared to evaluate arguments that will be the focus of the remainder of the course.

The purpose of an argument is to persuade someone or prove something. This being the case, there must be two parts to every argument.

1) the part you want to persuade someone of which is called the conclusion and
2) the part that provides the evidence which is the premise(s).

As we will see, arguments can contain one or more premises and some arguments can contain more than one conclusion. Though the arguments we analyze this semester will contain only one conclusion. What we need to be able to do is to distinguish these two parts of the argument.

There are several ways to do this but before we proceed we need to make a distinction between argument and opinion. This will be an important recurring distinction throughout the semester. Many people confuse these two but there are important differences. While arguments may contain opinions (these can be present in the conclusion) there must be more to an argument. An opinion is unsupported by evidence. An argument must present evidence (in the premises) to support the conclusion.

One of the easiest ways to distinguish between premises and conclusion is to look for certain indicator words within an argument. Certain words indicate the presence of the conclusion, while other words indicate premises.

The most common conclusion indicator words are: therefore, it follows that, accordingly, hence, it must be that, consequently, and so.

Use these words to find the conclusion as they are very reliable in their indication of the conclusion. Other words indicate the conclusion which are less common but it is a good idea to be familiar with at least the most common ones.

The most common premise indicator words are: since, because, for, and given that.

However, a word of caution is in order about premise indicator words. By and large, they are less reliable as indicators because they can have more uses than simply as premise indicators. For example, the word because. While this can indicate a premise the more common usage (in our text especially) is for it to indicate an explanation. We will be covering these in the next lecture. Just use caution when considering what the word "because" indicates. Also, the word "since" should be carefully considered. This is so because it has dual use as well. Since can indicate a premise but it can also indicate a passage of time. For example: "It's been so long since I've been to Florida I don't remember what it's like there." In this sentence, the word "since" is not indicating a premise at all. So, context is important.

One more thing to consider when distinguishing premises from the conclusion in an argument. What if the argument contains no indicator words? This can complicate things slightly but consider this. Given that the purpose of an argument is to persuade, it makes sense that the conclusion is reasonably easy to identify. One way to do this is with indicator words but another way is to place the conclusion first in the argument. In any case, to properly evaluate an argument, you first need to be able to identify the parts of the argument and understand what they do and how they relate to each other.

RECOGNIZING ARGUMENTS

We will now proceed to examine two methods for recognizing arguments. The first of these is to understand the components of an argument (premises, conclusion) which we have already looked at briefly. The second way is to distinguish arguments from several types of non-arguments.

Let's begin with considering two components every argument must have. The first is the factual claim. In arguments at least one of the statements must claim to present evidence or reasons. This occurs in the premises. The important thing to note here is that the premises don't have to present facts (after all they may be false premises) but they must claim to present facts. Secondly, the argument must make an inferential claim. Again, there must be a claim that the alleged evidence or reasons supports or implies something. This evidence, of course, is supposed to support or imply the conclusion. But, again, the important point here is that there doesn't have to be actual support given only a claim to provide support. This can be performed in two ways; either explicitly or implicitly.

An argument contains an explicit inferential claim if it makes the claim with indicator words; either premise or conclusion indicators. For example: Since today is Saturday, it follows that tomorrow is Sunday. Here, the inferential claim is explicit since I've used indicator words: "since" and "it follows that." On the other hand, the inferential claim may be expressed implicitly without using indicator words. Arguments that make such inferential claims can be more difficult to identify. However, once identified their conclusion should be easy to find. Remember? It almost always will be stated first.

Another way to identify such arguments is to distinguish them from non-arguments. We will identify three such non-arguments: conditional statements, explanations, and passages lacking an inferential claim. Let's begin with conditional statements. Conditional statements take the form "If A then B" where A and B can stand for any simple statements. For example, If today is Saturday, then we will not have logic class." Conditional statements, by themselves, can NEVER be arguments. I cannot overstate this. Conditional statements, by themselves, can NEVER be arguments. Why? Because they are not making the two claims that arguments must make. They can be interpreted as making an inferential claim; that is that one thing follows from another. However, they are not making a factual claim. In my example, I did not say that today was Saturday. I merely said, "if today is Saturday."

The interesting thing is that while conditional statements can never be arguments by themselves they can be parts of arguments as the examples in the text illustrate. So, just because a passage begins with the word "if" don't assume it's automatically a conditional statement. Read the entire passage. And by all means, if the passage contains conclusion indicator words don't ignore what these words are telling you!

The next non-argument we'll consider is the explanation. It may be obvious to state that explaining something is different from arguing for something. However, this will give us a clue as to how to distinguish explanations from arguments. The reason we need such clues is that it can be difficult to distinguish the two. Part of the reason for this is that they share similar forms. Arguments contain two parts; premises and conclusion. Explanations also contain two parts; explanans and explanandum. The explanandum is the part being explained and the explanans are the part doing the explaining.

To distinguish the two it is important to keep in mind the difference in intent. An argument is supposed to prove that something is true. An explanation is meant to explain why something is true. Now, think about this. In an argument does it seem to make sense to prove something true that we already know is true? Not likely. On the other hand, it makes perfect sense to explain why something is true given that we already know it. So to tell the difference between arguments and explanations begin by finding the main point of the passage in question (this will either be the conclusion or the explanandum). If the main point is something that needs proof or is not already known then the passage is likely an argument. On the other hand, if the main point is already known then it is likely the passage is an explanation.

Consider this example: "The Challenger spacecraft exploded after liftoff because an O-ring failed in one of the booster rockets." Now, the main point of this passage is "the Challenger spacecraft exploded after liftoff." Don't we already know this? Yes. Do we need someone to prove this to us? No. So this passage is an explanation.

DEDUCTIVE AND INDUCTIVE ARGUMENTS

The difference between deductive and inductive arguments is in how they claim to provide support for the conclusion. In deductive arguments, the premises are claiming to support the conclusion as a matter of necessity. Another way to think about this is that in a deductive argument the arguer is claiming that it is impossible for the premises to be true and the conclusion to be false. For example: "Either today is Saturday or Sunday. But, since today is not Saturday, it follows that today is Sunday." Here, the conclusion seems to follow as a matter of necessity from the premises. In other words, if the premises are true the conclusion must also be true. This is the nature of deductive arguments.

Inductive arguments, on the other hand, provide support to the conclusion as a matter of probability. Another way of thinking about this is that the arguer is claiming that it is improbable for the conclusion to be false assuming the premises are true. But, there is no guarantee in inductive arguments since the conclusion is never given necessary support. Even in the best of inductive arguments the premises only provide probable support to the conclusion. For example: "It's been snowing for six days straight therefore tomorrow it will snow." This conclusion may be highly probable but not guaranteed. This is the nature of inductive arguments.

To clarify this distinction, we can outline 5 deductive and 6 inductive types of arguments.

Deductive:
Arguments based on mathematics
Arguments based on definitions
Hypothetical Syllogisms
Disjunctive Syllogisms
Categorical Syllogisms

Inductive:
Arguments from authority
Arguments from sign
Arguments from analogy
Causal Inferences
Generalizations
Predictions

Deductive:

Arguments based on mathematics: since mathematics is a deductive science arguments based on mathematics are also considered deductive. However, two things to be aware of here. Just because an argument contains numbers doesn't necessarily mean it's deductive. Statistical arguments also use numbers but are considered inductive (they go under the generalization heading). Also, just because an argument does not contain numbers doesn't mean it can't be mathematical. Consider #8 in the exercises. No numbers, but it is deductive because it is based on a mathematical principle.

Arguments based on definitions: These arguments draw a conclusion from the meaning of a word. Since there is usually a necessary connection between a word and its meaning these arguments are regarded as deductive.

Hypothetical Syllogisms: when we discussed conditional statements we mentioned that they could be parts of arguments. These are the arguments that they are most often a part of. Syllogisms are arguments that contain three statements: two premises and one conclusion. Syllogisms can be either inductive or deductive but the three syllogisms listed here under the deductive heading are ALWAYS deductive. In this respect, form outweighs content. As long as a syllogism contains at least one conditional statement it will be deductive.

Disjunctive Syllogisms: These MUST contain a disjunction. What's that? An Either/Or statement. This must be stated explicitly. Don't simply interpret a statement as possibly being a disjunction. It has to say "Either one thing Or another" as in "Either today is Saturday or Sunday."

Categorical Syllogisms: Every statement in the syllogism must begin with a categorical word: all, no, and some.

<u>Inductive</u>

Arguments from Authority: These arguments proceed from the claim that an authority or presumed authority makes to the conclusion that we should accept that authority's claim.

Arguments from sign: These arguments proceed from the knowledge gained from a sign to conclude something about what the sign symbolizes. Here "sign" can mean any kind of message that can communicate information. These could be literal signs or labels but could also be other indications that could be sources of information.

Arguments from analogy: These occur when you compare two things that are similar in some respects. It is important to remember that analogies make explicit comparisons. If no comparison is being made it's quite likely the argument is not an analogy.

Causal Inferences: These arguments usually proceed from the knowledge of a cause to conclude something about the effect or from the knowledge of the effect to conclude something about the cause.

Generalizations: For an argument to be a generalization the conclusion must make a statement about many or all the members of a group. If the conclusion of an argument is about a specific person or object then it is very unlikely that the argument is a generalization.

Predictions: These can be related to causality but differ in that they proceed from an event in the past to draw conclusions about the future.

Something else to note about this list of arguments. This is not an exhaustive list. Some arguments don't fall into one of these eleven categories. For example, many scientific arguments are not clearly in any of these categories and so determining whether they are deductive or inductive can be difficult. In cases like this, you will need to examine the argument to determine whether the premises are attempting to provide necessary or probable support for the conclusion to tell what kind of argument it is.

For the bulk of this course, we will be looking at inductive arguments. This is because in ordinary discourse these kinds of arguments are by far the more common. Many arguments in social policy and other areas of life use inductive arguments such as predictions, causal arguments, generalizations, analogies, and arguments from authority. While deductive arguments do arise as well they are not nearly as common.

VALIDITY, SOUNDNESS, STRENGTH, AND COGENCY

We now turn to evaluating arguments. As we will see determining whether an argument is good or bad can be a technical process. Since we have two kinds of arguments (deductive, and inductive) we will need two methods of evaluation. And, since each argument must make two claims (factual, inferential) we must evaluate both claims to fully evaluate an argument. In both deductive and inductive arguments, we'll begin by examining the inferential claim.

The reason for this is simple. Since the inferential claim is where the arguer claims that the premises support the conclusion we need to determine whether this support is there independent of whether the premises are true. Another way to think of this is to ask whether it would make an argument good if the premises were true but didn't support the conclusion. The answer, of course, is no. The inferential claim establishes a connection between the premises and the conclusion. If there's no connection it really makes little difference whether the premises are true or false. So, it is only after we determine that there is a good inferential connection between premises and conclusion that we consider the truth of the premises.

Let's begin with **deductive arguments**. Evaluating the inferential claim in deductive arguments is a question of validity. It is important to note that validity has absolutely nothing to do with the actual truth value of statements in an argument. Validity and truth are two entirely different concepts. All we're concerned with in determining an argument's validity is whether the premises support the conclusion. To determine this involves a two-step process:

1. Assume the premises are true. They may or may not be true but begin with this assumption.
2. Based on this assumption ask the following question: Does the conclusion have to be true?

If the answer is yes, the argument is valid. If the answer is no, the argument is invalid.

Of course, we are concerned with truth as well. A really good argument needs to be valid and have true premises. The question of truth for deductive arguments is the question of soundness.

Here's the method for determining soundness. First, do not assume the premises are true; we only do that for the question of validity. Simply ask the following question:

Are the premises REALLY true? If the answer is yes, the argument is sound. If the answer is no, the argument is unsound.

Interestingly enough every invalid argument is, by definition, unsound. Also, it is possible that there be a third answer to the question posed above. That is, you may not know whether the premises are really true or not. So sometimes soundness cannot be determined. Consider this example: "I have seven pears on my kitchen table and I have five apples on my table. Therefore, I have twelve pieces of fruit on my kitchen table."

Clearly, this argument is valid (7+5=12). But can you determine whether the premises are really true? You shouldn't be able to since you don't know what I have on my kitchen table. So here soundness would be undetermined. In general, there are two cases where you may not know the truth of the premises. Cases like this one (where there's no reason to think you should be able to determine soundness) and cases where you may not know whether the premises are true or false (but you should). An example like this would be:

Since Moby Dick was written by Shakespeare, and Moby Dick is a science fiction novel, it follows that Shakespeare wrote a science fiction novel.

This is an unsound argument. You may not know who wrote Moby Dick, but you should.

We now turn to the method for evaluating **inductive arguments**. The method is similar to deductive arguments. We evaluate the inferential claim first. But since the connection between premises and conclusion is one based on probability we evaluate it differently. For inductive arguments, this connection is a question of strength. Notice that in deductive arguments the argument is either valid or invalid. There's no in-between. But for inductive arguments, the terms strong and weak are terms of degree implying that there could be some gray area. This is the nature of inductive reasoning and cannot be avoided.

The method for evaluating the inferential claim in inductive arguments involves two steps:

1. Assume the premises are true (just like in deductive arguments).
2. Based on that assumption ask the following question: Is the conclusion probably true?

If the answer is yes, the argument is strong. If the answer is no, the argument is weak. For an argument to be judged strong, the premises must provide probable support for the conclusion. If the conclusion is probably true independent of the premises then the argument is considered weak.

We also need to consider the truth of the premises and, for inductive arguments, this is the question of cogency. Again, we are interested in the real truth value of the premises so we do not assume the premises are true. This question only arises for strong arguments. All weak arguments are by definition uncogent. Like deductive arguments we ask the following question:
Are the premises REALLY true?

Also like deductive arguments, sometimes cogency cannot be determined. To say an argument is cogent means the premises really are true. To say an argument is uncogent means the premises are really false. But, what if you can't tell whether the premises are really true or false? Then cogency simply cannot be determined. Remember, all weak arguments are always uncogent.

SOMETIMES "I DON'T KNOW" IS THE RIGHT ANSWER

One of the most difficult insights for students to grasp about the concepts of soundness and cogency is that they cannot always be determined. I have noticed that when students evaluate arguments they sometimes feel the need to state that an argument is sound or unsound, cogent or uncogent when they cannot possibly have any means of truly determining this. Let's look at the concepts closer and try to understand why they sometimes cannot be determined.

The definition of a sound argument is one that is valid and has true premises. A cogent argument is strong, has true premises, and does not omit any premises that would entail a different conclusion from the one drawn in the argument. In valid deductive arguments, the determination that the argument is unsound simply means that the premises are false. In a strong inductive argument, the determination that the argument is uncogent simply means that the premises are false. Of course, an invalid argument is automatically judged unsound just as a weak argument is automatically uncogent.

With these parameters in mind let's look at some examples.

Since Moby Dick was written by Shakespeare and Moby Dick is a science fiction novel, it follows that Shakespeare wrote a science fiction novel. In this deductive argument, the premises (assumed true) necessarily entail that the conclusion is true so the argument is valid. Once we determine this we ask whether the premises are really true or false. In this example the premises are false so the argument is unsound. That should be easy enough.

But, consider this example. Since Agatha is the mother of Raquel and the sister of Tom, it follows that Tom is the uncle of Raquel. Again this is a valid argument so we need to determine (if possible) soundness. But, ask yourself the question: Are these premises really true? You might say yes, but how do you know? You might say we don't know so the argument is unsound. But wait! Judging an argument unsound simply means that the premises are false. Do you know this to be the case either? No. Here is a case where the truth of the premises cannot be determined so soundness cannot be determined. What would be the proper answer to this question then? We would say it is deductive and valid but the soundness cannot be determined. Sometimes in logic "I don't know" is the right answer!

This also occurs in inductive arguments. Consider this example. Coca-Cola is an extremely popular soft drink. Therefore, probably someone somewhere is drinking a Coke right this minute. Assuming the premise is true the conclusion is probably also true so it is a strong argument. Now, is the premise really true? Sure. So, the argument is also cogent.

But what about this one? Harry will never be able to solve that difficult problem in advanced calculus in the limited time allowed. He has never studied anything beyond algebra and in that, he earned a C minus. Here is another strong argument since if we assume the premise is true (that he never studied anything beyond algebra and in that he earned only a C minus) the conclusion is probably also true. But, when it comes time to assess cogency we no longer assume the premise is true. We need to determine whether the premise really is true. Is it?

Again, I would ask you to consider how you would know this. Do you know Harry? No. So, here is a case where cogency cannot be determined.

To this some will reply: Well, then soundness and cogency can never really be determined. But, we've already seen examples where we can easily determine soundness and cogency so this is not the case. My only point is that there are arguments where the truth of the premises cannot be established. That is to say, no one could know whether they are true or false.

Please note that this is different from the problem you may run into in some examples where you personally do not know whether the premises are true or false. If you personally do not know, that does not necessarily mean that soundness or cogency cannot be determined. It may simply mean that you cannot determine them. In practical terms, this may be a problem for you on the exam since I will expect that in cases where soundness or cogency can be determined, you can determine it. A few examples may clarify this point.

Every map of the United States shows that Alabama is situated on the Pacific coast. Therefore, Alabama must be a western state. This inductive argument is strong since the conclusion follows from the premise assuming the premise is true. Is it a cogent argument? You may not know and so you may think the answer is that cogency cannot be determined.

However, this is a case where cogency can be known (whether you know it or not). It is false that every map of the United States shows that Alabama is situated on the Pacific coast. It is quite likely that no maps show this (and you should know this as you should know where Alabama is really located!). So, this argument can be determined and is uncogent.

How about this one? The United States Congress has more members than there are days in the year. Therefore, at least two members of Congress have the same birthday. This deductive argument is valid since the conclusion necessarily follows from the truth of the premise. So, now we need to determine soundness. Can we? Yes. The premise is true so the argument is sound.

But, what is the difference between these examples and the ones above that were undetermined? Look at the example with Raquel and Tom. There is no reason anyone should be expected to know who these people are. How could you ever know this? This is why soundness cannot be determined. However, in the example about Congress, it is clearly knowable by someone and I maintain it ought to be known by everyone in the class as an educated person.

So, please be aware of these issues. Take care to note the cases where soundness and cogency can be determined but also be aware of cases where they cannot. In such cases recognize that there is no logical basis for determining soundness or cogency and refrain from doing so.

In other words, recognize when you don't know something and acknowledge that. As we'll see in the next lesson titled The Illusion of Knowledge, this is much more difficult than it seems.

THE KNOWLEDGE ILLUSION

The world around us is very complicated. But, while we tend to recognize this fact when prompted we routinely ignore it when it comes to expressing our own opinions on the world. We tend to recognize the limits of other people's knowledge while failing to recognize our own limits. We all suffer from what Steven Sloman and Philip Fernbach refer to in their book The Knowledge Illusion as the illusion of explanatory depth.

Consider the following example from their book:
On a scale from 1 to 7, [1 meaning not at all and 7 meaning completely], how well do you understand how zippers work?
How does a zipper work? Describe in as much detail as you can all the steps involved in a zipper's operation.

If you're like most people, you rated your understanding of question 1 as very high. But, if you're like most of those people you discovered in step two that you were virtually incapable of explaining in any sort of detail how a zipper actually worked.

This is the knowledge illusion. As they point out in the book, "Our point is not that people are ignorant. It's that people are more ignorant than they think they are. We all suffer, to a greater or lesser extent, from an illusion of understanding, an illusion that we understand how things work when in fact our understanding is meager."

Please understand that I'm not sharing this with you to make you feel bad. I'm sharing this to point out a crucial insight that will help us deal with real-world issues in this course (and help you with those issues outside of this course) and to encourage you to cultivate an important critical thinking disposition: intellectual humility.

What does any of this have to do with a course in logic? Well, take the example above about the zipper. Now, instead of asking about how a zipper works, substitute some issue requiring clear thinking and evidence: abortion, capital punishment, immigration, cloning, etc.
See the problem?

If you're like most people, you have an opinion on these issues. You think your opinion is well-founded and backed up with knowledge and facts.

You're also suffering from the illusion of explanatory depth. You think your knowledge of the issue is deeper than it really is. Don't believe me. Try it.

Consider the contentious (and because of what is currently happening in Texas very relevant) issue of abortion. Set aside your position on abortion for a moment and consider just how much you know about this issue.

What are the three most important Supreme Court rulings on abortion?
What is the central argument in Roe v. Wade and what does it specifically imply about abortions?
What percentage of women each year who become pregnant have an abortion?
What is the breakdown in terms of trimesters for abortion? How many in each trimester?
What laws are states able to pass regarding abortion?

Are you able to answer each of these questions in sufficient detail?
And these are just a few of the questions you'd need to have answers to really claim you have a depth of knowledge on the abortion issue.
If you are unable to answer these questions, then what exactly is your stance on abortion (whatever it is) based on?

Now, consider all of the other issues we could consider in this way. Each one entails a depth of knowledge that likely, most people do not have. But, most of those people have strong opinions on each of those issues.

So, what should we do about this problem? There are several options to consider.

1. Start now and learn as much as you can about all of these issues.
The trouble here, of course, is that you simply don't have enough time to do this. No one does. There's too much to master and too little time.

2. Continue to do what you've always done.
Have your opinions and express them. What difference does it make if you know or not? It's what you feel about these issues that matters.

The problem here is that it does make a difference. What if you're wrong and your push for a solution to the issue is a push towards action that just makes things worse?

3. Drop out of the game altogether.
Don't have opinions and if you do have them, don't express them. Don't participate in civic life. Don't vote.

The problem with this choice is that we need people to participate in civil society. In a participatory democracy, a representative republic, we need people to vote, advocate, inquire, and be a check on politicians. People need to be able to hold their representatives accountable for their actions.

4. Cultivate intellectual humility. Learn what you can but recognize that you won't be able to be an expert on all of these issues. Also, recognize that you share the same problem with everyone

else. Encourage others to see the limits of your knowledge are the same as the limits of their knowledge.

As Julia Galef advocates in her book The Scout Mindset, hold your beliefs lightly. Don't be so sure of your own beliefs that you can't or won't listen to others. Be a person that works for dialog and discussion. See the common ground between you and those who hold different opinions. Of course, you can express your opinions and argue for them passionately. But, be prepared and open to seeing other views and seriously consider them as well.

UNIT TWO: RULES OF REASON

LAWS OF THOUGHT

Logic is based on the presumption that there are some very basic, self-evident laws of thought that ought to guide our thinking. These laws help us understand what counts as a good argument, what counts as good evidence, and what inferences should be accepted and rejected. In a sense, everything we do in logic follows from these laws.

The laws themselves, which I will discuss below, are self-evident descriptions of how the world works. To say they are "self-evident" means that we don't need any other evidence to know they are truly beyond our understanding of what they mean. This raises an interesting point. If you don't believe these are self-evident, that is, if you don't believe the claims that these rules are making are true, then there's very little that can be done to persuade you otherwise. In some sense, the conversation won't be able to proceed without your agreement with these rules.

There are three laws of thought we should consider:
The law of non-contradiction: Something cannot both be and not be at the same time and in the same respect.
The law of identity: Everything is identical with itself.
The law of excluded middle: For any given property or attribute, everything either has that property or does not have that property.

You may be thinking that there are many exceptions to these rules but they are probably not exceptions at all just misunderstandings of what the rules imply.

For example, you might say that something can be and not be. It could be raining one minute and not raining the next. But, this is not an exception at all. The rules do not say that things cannot change or that one thing (or state of affairs) could not become a different one. It is stating that one thing cannot be that something and not be that something at the same time and in the same respect. It's this last qualifier that is critical. A person can be both a parent and daughter without violating the rule since the person's state as a parent does not preclude there also being a daughter. What they cannot be is both a parent and not a parent at the same time and in the same respect.

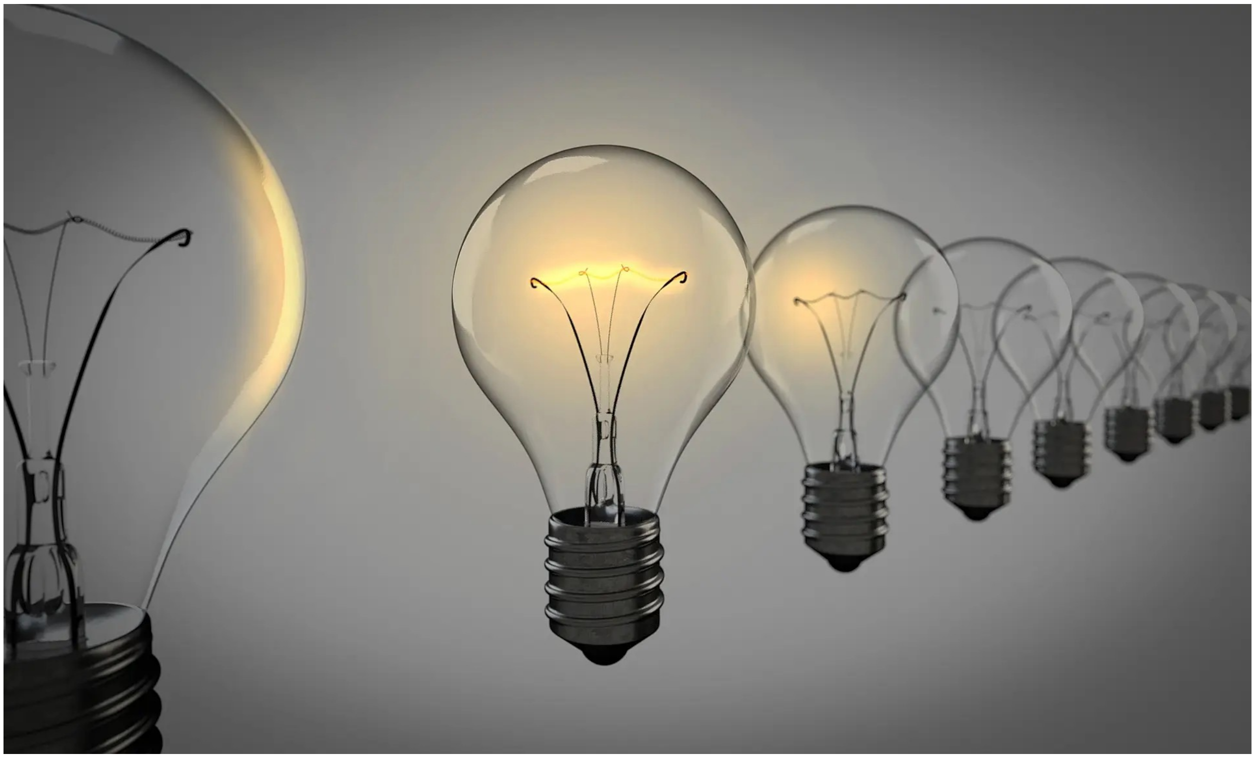
Once you understand this it should be clear that the rule is asserting nothing more than how things exist. To the extent that things seem to violate this rule, it is most likely that the violation is based on our lack of understanding regarding how the world works, not the inherent falsity of the law of thought. (Confused yet? I hope not!)

If you consider what the other two laws are saying it should become clear that they are just slightly different descriptions of the law of non-contradiction. If something cannot both be and not be at the same time that is the same as saying that something is identical to itself. This can be proven in formal logic (a subject we'll consider in another lesson). The same goes for the law of excluded middle. This can be shown with a simple example:

If it cannot be both raining and not raining in the place and time then, for any given time and place, it must either be raining or not raining.

What these laws of thought provide is a foundation on which we can build other more interesting and useful rules of reason which provide us with a framework for evaluating claims and theories about how the world works. This is a central feature in the work of logic. Ideally, for any given theory we should be able to provide a method for determining whether that theory is the best possible explanation for all of the evidence we have available to us. Let's consider this aspect of logic next.

THEORIES AND EVIDENCE



How do we know that the available evidence is sufficient to warrant accepting any given theory? There are criteria that can be used to evaluate a theory. What makes these criteria useful is that they allow us to determine whether our theories really do what they claim to which is to provide us with useful explanations for how the world works. A good set of criteria is outlined in How to Think About Weird Things by Theodore Schick and Lewis Vaughn. There are five elements to evaluate any theory.

First, a theory should be **testable.** If you cannot even figure out how to go about determining if your theory explains the evidence, you don't have a good theory. To be testable means your hypothesis "predicts something more than what is predicted by the background theory alone." In short, we need this criterion because if there's no way to tell whether a theory is true or false it's really no good to us.

Second, a theory should be **fruitful**. What this means is that a good theory should make novel predictions. It should not only account for the evidence at hand but be able to address evidence that comes in later and even predict such new evidence. Einstein's theory of relativity is a good example of a fruitful theory because it made the novel prediction that light would be visible from a star behind the sun. After all, the light would be bent by the gravitational field around the sun to be visible on earth. And, concerning criterion number one, this was a testable claim. Once tested, it was verified.

Third, a theory should have a **wide scope**. That is, a good theory explains a wide field of evidence. One of the differences between theories and hypotheses is their scope. Hypotheses address specific questions whereas theories attempt to provide a broad explanatory device. Theories that can explain a wide array of things are preferred, other things being equal, to more narrow theories.

Fourth, a theory should be **simple**. This term should not be confused with simplistic. Many scientific theories are complex in terms of our ability to understand them but simple in the sense that they postulate fewer underlying entities or assumptions. A good example is the difference between Copernicus and Ptolemy. Ptolemy's geocentric theory could explain the orbits of the planets but it was quite complex whereas Copernicus' theory explained the same observable phenomena with less complexity. So, other things being equal, that theory was the better theory.

Think of it this way. Suppose I come up with a theory to explain how the lights in my housework but it involves little gremlins running inside the light bulbs. Someone else can explain the same phenomenon but without postulating gremlins. So, their theory is simpler than mine. It should also be pointed out that my gremlin theory may fail on other criteria as well such as being testable.

Finally, a theory should be **conservative**. Not in the political sense of the word. Rather, it should fit in with other things we know. If we have an explanation for something that we think is fairly certain and accurate then a new theory should fit in with that prior explanation. If it doesn't fit that may indicate our prior knowledge is flawed. We have to be open to that possibility but the burden of proof is on the new theory. An interesting examination of how this process works is offered in Thomas Kuhn's book The Structure of Scientific Revolutions.

Many of you will be inclined to lodge the following criticism at this point: But, that’s "just a theory." The criticism here is supposed to be that any given theory is not an established fact. But, this misunderstands the relationship between fact and theory. No theory is a fact because facts and theories are two different things entirely. The facts are what we observe about the world around us. But, these facts need an explanation. This is what a theory is designed to do. It is a well-formulated attempt to explain the facts we observe. So, we observe the motion of the planets and the fact that an apple falls to the ground when we drop it. The theory of relativity attempts to explain these things.

We observe different species and varieties of animals in the natural world and the theory of evolution attempts to explain how these varieties arose. We observe the motion of subatomic particles and the theory of quantum mechanics attempts to explain these observations. In each case, we begin with observations and construct an explanation to account for them. In each case, it makes little sense to criticize the theory by saying it's not a fact. Of course not! Theories are not facts and do not attempt to be. Theories can be correct or incorrect and the criteria outlined above is the best way to determine this. But, you must also understand what a theory is attempting to do.

MORE RULES OF REASON

The laws of thought we just discussed are the foundation for more common rules of reason. Some of these are the basis of arguments you use almost daily. To show you just how common (and hopefully commonsensical) these rules are I will provide you with just the premises and you should be able to supply the conclusion of each one.

1. **Modus Ponens** (the affirming mode). This rule allows us to infer a conclusion from the following premises. Let's see if you can derive the conclusion.

If today is Saturday, then we will go swimming. Today is Saturday. Therefore?

2. **Modus Tollens** (the denying mode). Here are the premises. See if you can provide the conclusion.
If you can beat our prices, then pigs can fly. Pigs cannot fly. Therefore?

3. **Hypothetical Syllogism** [A syllogism is a three-step argument: two premises, one conclusion]. This one works like a chain of reasoning.
If this animal is a cat, then this animal is a mammal.

If this animal is a mammal, then this animal gives birth to live young.
Therefore?

4. **Disjunctive Syllogism**. Either today is Wednesday or Thursday, Today is not Wednesday. Therefore?

How are you doing?

Did you derive the following conclusions?

1. We will go swimming.
2. You can't beat our prices.
3. If this animal is a cat, then this animal gives birth to live young.
4. Today is Thursday.

OK, one more common rule of reason.

5. **Constructive dilemma**. Here the form is a little more complex. See if you can derive the conclusion.

If we choose nuclear power then we increase the risk of a nuclear accident. But, if we choose conventional power then we add to the greenhouse effect. We must either choose nuclear power or conventional power. Therefore?

Notice, that these rules merely lay out what conclusion must be derived given the premises. The conclusion follows as a result of the form of the argument, not the truth of the premises. That is, the premises may be false but assuming they are true the given conclusion will follow.

This sometimes leads to confusion among students struggling to learn the details of formal logic. How can an argument be good if the premises are false?

Well, the answer is that there are two parts to arguments (we addressed this in the lesson on the basics of logic) and formal logic is only evaluating one part of the argument; the part expressed by the form. So, just because an argument's form is correct doesn't necessarily mean it is a good argument. A good argument will have a correct form and true premises. The question of truth is one not usually addressed in formal logic since the focus is on assessing the form.

We will be returning to these argument forms in Units Five and Six where we will be discussing symbolic logic.

THE LOGIC OF RELATIONS

Another aspect of formal logic (i.e. the study of how argument forms work) deals with the logic of relations. This involves how different groups of items are related to each other. Categorical Logic is an abstract system designed to make understanding the logic of relations easier to deal with. Let's think about why this is important. David Levy once said, "one of the most fundamental and pervasive of all human psychological activities is the propensity to categorize." Just think about how we think of people in terms of categories:

Democrats, Republicans, Terrorists, Soldiers, Employees, Employers, Students, Teachers, Scientists, Enemies, Allies, The faithful, The unemployed, The homeless

You get the idea. What categorical logic allows us to see is that the way these groups relate to each other is governed by a set of systematic rules. In this respect what logic is really doing is providing a coherent organized description of the world we inhabit. This is important because it allows us to deal with the complexities of our world more easily.

But, formal logic often leads to a confusion. Many students wonder whether they are really expected to take the claims and arguments that people make and translate them into categorical or propositional logic before analyzing them. This seems so odd that it's hard to believe that even logicians do this. In other words, the skills that we teach in formal logic don't look like skills anyone applies. Well, in one sense that is true. No one expects you to stop in the middle of an argument to analyze it in categorical terms. What formal logic encourages though is a sense of what is logical. Once you have mastered the formal rules you begin to get a "feel" for what counts as a good argument. Once you've developed this logical sense you no longer need to go through the formal process every time you analyze an argument.

In this respect learning formal logic is like learning to read music or read a map or read words! As you learn these skills it takes time to apply them and you have to focus on the mechanics. But, once you master the skill you can simply navigate or play music or read poetry without focusing on the mechanics. This is your ultimate goal in logic. You will need to work to internalize these skills but once you do you'll be able to use your logical sense without thinking consciously about it. But to use the rules you first have to learn them!

Let's consider a few of these rules. First, we need to outline a few basics of categorical logic. Categorical statements come in one of four forms:

All S are P
No S are P
Some S are P
Some S are not P

In logic, they are often abbreviated with letters as follows:

A: All S are P
E: No S are P
I: Some S are P
O: Some S are not P

In each of these cases, the terms S and P can stand for any noun or noun phrase. So, All S are P could stand for "All cats are animals." If we plug in the terms "cats" and "animals" into the other three statements we'll get:

No cats are animals.
Some cats are animals.
Some cats are not animals.

You can instantly recognize which of these four statements are true and which are false. So, there's nothing very interesting here. Not yet.

But, suppose you have a set of statements with terms you are unfamiliar with. In such a case you won't instantly know which ones are true and which are false. But, with a few basic rules of categorical logic, you can deduce which statements are true and which are false and other useful information even in cases where you don't know what the terms refer to. Let's look at how this works beginning with the statement "All empiricists are phenomenologists."

THE TRADITIONAL SQUARE OF OPPOSITION

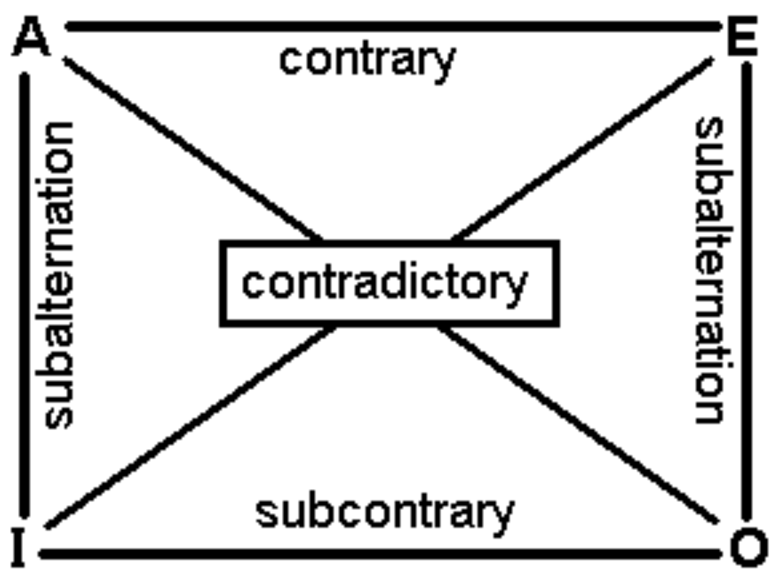
So, “All empiricists are phenomenologists.” What does that even mean? Well, the point of what we’re about to do is that you don’t have to know what it means to make some useful deductions. To simplify things I will abbreviate the terms with their first letters so:

All E are P

Now, suppose All E are P is true. We can deduce the following things about the other categorical statements:

No E are P must be false.
Some E are P must be true.
Some E are not P must be false.

How do we know this? Through a set of rules called the “traditional square of opposition.”



Remember the letters stand for statements as follows:

A: All S are P
E: No S are P
I: Some S are P
O: Some S are not P

Consider the following categorical statements:

All cats are animals.
No cats are animals.
Some cats are animals.
Some cats are not animals.

Now, the truth value of these four statements should be obvious. The A statement (All cats are animals) is true. The E statement is false, the I statement is true, and the O statement is false. Of course, sometimes the truth values are different. For example:

All dogs are cats.
No dogs are cats.
Some dogs are cats.
Some dogs are not cats.

Now, the A statement is false, the E is true, the I is false, and the O is true. But is there any pattern to these truth values? What happens if we don't know the truth value of the statements? Is there anything we can say about these statements if we only know the truth value of one of them? These questions can be answered by the traditional square of opposition.

Let's begin with the two universal statements (A, E).

The question we need to ask here is can they ever both be true? In other words, no matter what they are talking about, can there ever be a case where the pair are true? The answer is no. The reason is that in both statements the subject term is distributed. So, we're saying something about every member of that term. Given this, how can we say that all of the members of the subject term are part of the predicate term (A) AND at the same time say that every member of the subject term is excluded from the predicate term (E)?

Clearly, we can't so the rule that governs these two statements simply asserts that the two universals cannot both be true. From this, we can deduce that at least one of these two statements must be false. This is the rule of

contrary: At least one must be false; they cannot both be true.

With this rule, we can make inferences related to the universal statements. For example, if we know that the statement "All existentialists are phenomenologists" is true we can determine the truth value of the corresponding E-statement even if we don't know what "phenomenologists" or "existentialists" are. This is the power of these logical rules. We can make these inferences even if we don't know what the content is at all. This is why we will consider many examples with variables such as:

No S are P is true
therefore All S are P is: ?

False, right? Now, to complicate things slightly let's consider this example:

All S are P is false
therefore No S are P is: ?

You might be tempted to think that the answer is true but remember what the rule of contrary says.

At least one must be false, they cannot both be true.

But since we know All S are P is false we can't be certain whether No S are P is true or false since both truth values correspond to the rule. In the absence of any other information, we have to say that the truth value of No S are P in this example is undetermined.

Be sure you understand this before moving on because this will come up with the other rules as well.

The next rule we'll consider deals with the two particular statements: Some S are P and Some S are not P. The question here is can they both be false? Again, the answer is no. Remember, in logic "some" means there is at least one. However, if both statements were false there would be no subject term.

So the rule, called **subcontrary**, states that they cannot both be false which implies that at least one must be true. Like contrary, we can use this rule to make inferences regarding the particular statements. And again, like the rule of contrary sometimes the truth value cannot be determined.

Subcontrary: At least one must be true; they cannot both be false.

The two other rules deal with how the particular statements relate to the universals. One rule addresses this relationship in cases where the quality is the same (**subalternation**) and the other rule addresses this relationship when the quality is different (**contradictory**).

Let's consider the rule for the universal and particular of the same quality: **subalternation**. An example may help clarify this rule. Suppose you were thinking of attending a different college in a different state; one you've never been to but I have information about. So, I tell you that at this particular school All students are male. Now, can you tell what the truth value of the statement "Some students are male" is? True, right? Of course.

If all the students are male then any one of them must also be. However, suppose I was to tell you that it was true that Some students are male. From this information alone could you tell me whether it was true or false that All students are male? You shouldn't be able to determine the truth value of this statement. This is essentially what the rule of subalternation asserts. If you know the universal statement is true then you can also determine that the corresponding particular statement is also true. However, if you know the particular is true you cannot determine the corresponding universal. This rule is the same for the negative statements as well.

But there's another part to the rule. To illustrate this consider this example. Again, referring to the school you're thinking of attending I tell you that it's false that "Some students are not chemists." Now, can you tell me the truth value of "No students are chemists?" It has to be false, doesn't it? So this part of the rule asserts that if you know the particular statement is false the corresponding universal must also be false.

But again, there's the potential for undetermined truth values. If you know the universal is false, there's no way to determine what the truth value of the corresponding particular is. For example, if you know that "No students are male" is false you cannot be sure that "some students are not male" is also false.

Subalternation: Truth flows down; Falsity flows up.

The final rule on the square is **contradictory** and it's the easiest of the rules to remember and apply. This rule addresses statements that are opposite in their quantity and quality so it seems reasonable that they would also have opposite truth value. So, the A and O statements are contradictory as well as the E and I. If you know one is true the other must be false and if you know one is false the other must be true.

Contradictory: Statements have opposite truth values.

So what can we do with these rules? If you are given the truth value of one statement you should be able to use the square to determine the truth value of the other statements; keep in mind that sometimes you will get undetermined truth values. A little hint here might help. Undetermined truth-values will always come in pairs and they will always be contradictory to each other.

So, if you determine that one statement is undetermined its contradictory will also have undetermined truth value.

We can also use these rules to determine the validity of arguments that use the square. Basically, an argument is judged valid if the argument follows the rule and the argument is invalid if it is saying more than the rule will allow. Any argument in which the application of the appropriate rule yields an undetermined truth value is invalid.

QUANTITY, QUALITY, AND DISTRIBUTION

As we have seen, in categorical logic we have four statements:
All S are P
No S are P
Some S are P
Some S are not P

Each statement contains four parts. Let's begin with the letters S and P. These are abstract variables (called terms) that can denote nouns or noun phrases to describe the class of objects we are discussing. The first term in the sentence is called the subject term and the second term is the predicate term.

The four statements also contain two other parts. The first part of each sentence (all, no, or some) is called the quantifier. As we will see in a moment, the quantifier tells us the quantity of the statement. The sentence also contains a verb (are, are not). This is the only verb used in categorical logic. And these are the only four statements used. We do not use the statement "All S are not P" nor do we use the statement "No S are not P." Both of these are redundant and the second one is a double negative.

Each statement can be classified with two attributes: quantity and quality. Please remember these attributes as we will use them to determine validity both in this chapter and the next one. We can determine the quantity of a statement by looking at its quantifier. There are two types of quantity: universal and particular. Statements are universal if they assert something about every member of the subject term. So "All S are P" and "No S are P" are universal. "Some S are P" and "Some S are not P" are particular. In logic, the word "some" has a specific meaning, namely, "there is at least one." So for example when I say "Some cats are animals" what I mean is that there's at least one cat that's an animal.

Quality refers to whether we say something inclusive or exclusive. If we make a statement that one or all of the members of one class are included in another class then we're making an affirmative statement. So, "All S are P" and "Some S are P" are both affirmative. On the other hand, if we are making a statement that one or more members of one class are excluded from another class then we're making a negative statement. So, "No S are P" and "Some S are not P" are both negative.
So to summarize:

Statement	Quantity	Quality
All S are P	Universal	Affirmative
No S are P	Universal	Negative
Some S are P	Particular	Affirmative
Some S are not P	Particular	Negative

Notice the symmetry. Two universals, two particulars, and for each one affirmative and one negative.
The third attribute is different from the previous two. Distribution is not an attribute of statements but rather of the terms within the statements. So we say that a term within a statement is either distributed or not. Having said that here's what distribution means. A term is said to be distributed if we say one thing about every member of that term. This is a little confusing so some examples might help.

Consider the first statement: All S are P. Imagine the example All cats are animals. In this statement, we seem to be saying something about EVERY member of the first term: cats. So, the term cats is distributed. Now, here's the interesting part. For every statement having the form "All S are P" the subject term is distributed. But, we're not saying something about every animal in the statement so the term animals is NOT distributed.

The second statement: No S are P is different. For example, let's say No cats are dogs. Here we seem to be saying something about every cat so that term is distributed. However, we're also saying something about every dog as well so that term is also distributed. Again, this rule holds for every statement having the form "No S are P."

For the statement Some S are P let's use the example I mentioned above: Some cats are animals. Again, remember that "some" means "there is at least one" so the statement is true. (However, whether or not a statement is true does not affect distribution.) So, the statement is telling us that there's at least one cat and it's an animal. It seems that we're only talking about one cat here; the one cat that's an animal. So, neither term is distributed.

Finally, we have the statement Some S are not P. This one may seem a little odd. As with Some S are P the first term is not distributed since we're only talking about one of them. Let's use the example "Some cats are not animals." But, while we're only talking about one cat, we're talking about ALL of the animals. What are we saying about them? We're saying that ALL the animals are not that one cat. So the term "animals" is distributed. And, again this holds for every statement having the form "Some S are not P."

So to summarize:

Statement	Quantity	Quality	Distribution
All S are P	Universal	Affirmative	S is distributed
No S are P	Universal	Negative	S & P are distributed
Some S are P	Particular	Affirmative	neither S nor P
Some S are not P	Particular	Negative	P is distributed

One way to remember the distribution of these statements is to remember that universal statements distribute subject terms and negative statements distribute predicate terms.

As we'll see in the next lesson, these concepts are the keys to easily determining the validity of any categorical syllogism.

OTHER APPLICATIONS

There are two other applications of these logical operations that we should investigate. Essentially what we now have with the four rules on the traditional square and the three rules of equivalence are seven rules of inference that allow us to judge, within categorical logic, whether arguments are valid or invalid. Up until now, we've been using them separately. Let's see what happens when we use them together. Let's consider this argument:

It is false that some jogging events are not aerobic activities.
Therefore, it is false that no jogging events are aerobic activities.

To see which rule is being used it might help to look at the form of this argument without the content. It would look like this:

F) Some J are not A

F) No J are A

Looking at the arguments this way reminds us that validity is solely a function of form. We don't need to know what the content is to determine validity. In fact, it's easier to determine validity without looking at the content as it tends to distract us.

So, which rule is being used here? How can you tell whether it's one of the rules on the traditional square or an equivalence rule? Here's an easy hint. If something about the terms is changing (their position, term complement) it must be one of the equivalence rules (conversion, obversion, contraposition).

On the other hand, if NOTHING about the terms is changing, it has to be on the square. So, this example must be on the square since the terms are the same from premise to conclusion. Which rule? The premise is an O statement and the conclusion is an E statement so that's subalternation. Now, the question is whether it's valid or not. Is the rule being used correctly? Yes, so the argument is valid.

One more example:

All Barbie dolls are toys that engender a false sense of values.
Therefore, no Barbie dolls are toys that engender a true sense of values.

Again, we'll look at the form to help simplify this argument:

All B are F

No B are non-F

Notice something about the terms changes in this argument from premise to conclusion so it has to be one of the equivalence rules. Which one? Obversion. And since all obversions are valid this argument is valid.

The second thing we can do with the rules is a little more complex. Here we'll be using the rules in tandem instead of one at a time. To illustrate the basic principle of this consider this example:

All S are P

Some S are P

Is that a valid inference? The answer is yes because of the rule of subalternation. Now, take the conclusion of that argument and use it as the premise in the following inference:

Some S are P

Some P are S

Is this a valid inference? Again, the answer is yes due to the rule of conversion. Now, here's the payoff. Is the following a valid inference?

All S are P

Some P are S

Notice, that it contains the premise of the first example and the conclusion of the second. But the two examples had one statement in common (some S are P). If the first two inferences are valid, then this inference should be valid as well. Our job is to prove its validity by using the rules of inference together.

The easiest way to think about arguments like this is that you will need to turn the premise into the conclusion using the rules to do so.

So, to deduce Some P are S from All S are P we simply change All S are P to Some S are P (and the rule of subalternation justifies this inference). Having deduced Some S are P we can simply convert it to get the conclusion. What we need is a simple method to solve these problems. To illustrate this method let's look at another example in symbolic form:

Premise: No non-G are E

Conclusion: Some non-E are G

To prove this valid we need to show the steps it will take to turn the premise into the conclusion. Since we're thinking about this process as making changes to statements let's associate a specific rule with a specific change as follows:

Conversion: changes position of S and P terms
Obversion: changes 1 term complement in a statement
Contraposition: changes 2 term complements in a statement
Subalternation: changes the quantity of a statement
Contradictory: changes the quantity of a statement and its truth value
Contrary: changes truth value (for universal statements)
subcontrary: changes truth value (for particular statements)

So, we'll look at the problem and see if we can note any of these changes that occur from premise to conclusion. In this example, we have the following changes:

quantity (premise is universal, conclusion is particular)
2 term complements (non-G changes to G and E changes to non-E)

How do we change quantity? Subalternation
How do we change 2 term complements? Contraposition

So we need to do these two operations and we will have proved the argument valid. By the way, all the arguments in this section are valid it's just our job to prove them valid.

Premise: No non-G are E

Some non-G are not E (subalternation)

Some non-E are not G (contraposition)

Notice that the last statement in my proof is identical to the conclusion. That's what shows that the argument is valid. And that's all there is to it.

CATEGORICAL SYLLOGISMS

We now turn our attention to categorical syllogisms. Remember from chapter one that a syllogism is an argument with two premises and one conclusion. Categorical syllogisms contain three categorical statements. Here's an example:

All S are M
Some M are P

No P are S

Notice that there are three terms in this syllogism. Every categorical syllogism must have three terms; one is in both premises (in this case M) and this term is called the middle term. The subject term of the conclusion (in this case P) is the minor term and must occur in the minor premise; which is the second premise. The predicate term of the conclusion (S) is the major term and must occur in the major premise which is the first premise. A syllogism with three terms in the proper place is said to be in standard form.

Now, we need to determine whether the syllogisms are valid. To do this we will appeal to five rules of validity which are based on the three concepts of quantity, quality, and distribution. A syllogism can only be judged valid if it does not break any of the five rules. A syllogism is judged invalid if it violates any one of the first four rules. Finally, a syllogism is judged conditionally valid if it only violates rule five (we'll explain this in a moment). Here's a summary of the rules:

1. The middle term of a syllogism must be distributed. Since the middle term connects the major and minor terms we must assert something about every member of that term otherwise we can't establish any valid connection in the conclusion.

2. If a term is distributed in the conclusion it must be distributed in the premise as well. This is the syllogistic version of the rule of subalternation. You can't say something is true of all from the fact that something is true of the part. If the conclusion asserts something about every member of a certain term that assertion must be backed up with a similar assertion in the premise.

3. Two negative premises are not allowed. If the premises assert that the terms are separate, then you cannot validly assert that they are connected in the conclusion.

4. If the conclusion of the syllogism is affirmative, the premises cannot be negative. If the conclusion of the syllogism is negative, then one premise must be negative. This carries through the logic of rule three to the conclusion. A negative conclusion asserts that some or all of one term is separate from the other and that assertion must be supported by a premise that asserts something similar. An affirmative conclusion asserts a connection that must be backed up in the premises.

5. If the conclusion is particular, then one of the premises must be particular as well.

If an argument violates only this rule it may still be a valid argument. Remember that the word "some" means "there is at least one." So, it's implicitly making a statement about existence. Universal statements make no such statement about existence. If I say "All unicorns are mammals" I'm relating two classes of objects without asserting that there are any unicorns. However, if I say "some unicorns are mammals" I am making an implicit statement that unicorns exist. The main idea in this rule is that if the terms refer to things that exist the argument will be valid; otherwise, the argument is invalid. Except that many times we won't know what the terms refer to so we can only say that the argument is conditionally valid. For example:

No S are M
All M are P

Some P are not S

This argument violates only rule number 5 and so is conditionally valid. Obviously, if the argument violated rule 5 and some other rule the argument would be invalid (because of the other rule is violated).

UNIT THREE: FALLACIES

BARRIERS TO GOOD CRITICAL THINKING

"We think in logic as we talk in prose without aiming at doing so."
John Henry Newman

While this is a true statement we do have to remember that even as we have to learn to talk and practice at it, we also have to learn to think and practice at it as well. This is the point of studying logic. Most people when asked will self-report that they are logical thinkers. Psychologists refer to this as attribution bias. We often attribute to ourselves more abilities than we attribute to others. And we are notoriously poor at self-assessment. So, even if you already think you are a logical person, you may still benefit from studying logic. What could it hurt to hone your skills further?!

There are many barriers to good logical thinking. Just consider the following list from a Critical Thinking textbook:
lack of relevant background information

poor reading skills
poor listening skills
prejudice
superstition
relativistic thinking
distrust in reason
short term thinking
selective perception
selective memory
unwarranted assumptions
stereotyping
narrow-mindedness
close-mindedness

To become a good critical thinker, one must overcome these barriers. The presumption of a course in logic is that one of the tools to overcome such barriers is the study of the process of reasoning: logic! At times our study of logic will seem irrelevant to real-life situations. It is my intention in these essays to show that logic is practical and can be applied in real life. I hope you'll feel free to engage with these readings and share your comments and questions regarding them. Logic is best done in dialogue so don't let me just lecture at you! Let's discuss logic and its usefulness together.

We all suffer from time to time from these barriers to good thinking. A famous example is related in the book Thinking by Gary Kirby and Jeffery Goodpaster:

“The opposite of clear thinking is confusion, and it can lead to costly conclusions. A young American inventor appeared before Napoleon and offered him a means to defeat the British navy: a ship that could sail against the wind and waves and outmaneuver the British fleet. Napoleon scorned his offer, called the American a crackpot, and sent him away. That young man was Robert Fulton. Napoleon had just turned down the steamship.

“Napoleon’s thinking error was common to most of us: He was blinded by the past. In addition, he was blinded by his quick temper.”

As we’ll see through the course of this semester, logic can function as the antidote to such problems as close-mindedness, irrational anger, superstition, and prejudice. However, logic cannot be the antidote to all the barriers to good thinking. The importance of relevant background information is critical to good reasoning. But, logic doesn’t focus so much on this. Rather, the focus is on more mechanical aspects of our reasoning process. While these are more difficult to apply they are practical and I hope to show through their practical value.

For now, let me point out with a powerful example something I mentioned in my lesson Why Study Logic. The world is filled with people who are trying to persuade you to believe in something, to vote for someone, to buy something. They often resort to trickery and deceit to succeed. And yes, you’ve guessed it my point is that logic is your defense against such trickery.

The example I want to share with you comes from Malcolm Gladwell’s best-selling book The Tipping Point. In it, he relates a psychological study that showed that students could be persuaded simply by being encouraged to nod their heads as they listened to an editorial about raising tuition. Now, I’m sure this example hits close to home which is why I’m using it. Of course, it’s not just students who are susceptible to such influence. Everyone is to some extent.

What logic does is force us to look at our mostly unconscious ways of thinking consciously.

It’s a lot like looking at the engine in your car. If you’ve never seen the engine it looks strange and incomprehensible. But, the engine is the means by which your car runs. This is what I mean by saying that logic will examine the mechanics of thinking; the rules we use to reason. This will take practice because at first, the rules of logic will look unfamiliar. This is because we’ll be looking at them apart from their ordinary context. But, this is the best way we have for examining them, analyzing them, and improving our use of them. I hope you’ll find that the payoff is worth the effort!

CONSISTENCY

A central value in logic is consistency. This is when a collection of statements can all be true at the same time without any contradictions. In a set of consistent statements, there will not be any that can be shown to be false when others are known to be true. Stated this way, it doesn't sound like a very interesting thing or even very useful but let's look at it closer to figure out why consistency should be valued.

Any set of statements can be tested for consistency but things get really interesting when we look at specific sets of statements. For example, like the set of beliefs you hold.

Ideally, this set of statements (your beliefs) should be consistent. Why? In logic, we base our inferences on reason and evidence. So, that limits what we can infer from any given set of statements. But, if the statements are inconsistent absolutely anything can be inferred from them. This is because a set of inconsistent statements includes some which are false among others that are true. That removes any limits from what can be inferred from that set of statements.

The problem with this is quite simple. If anything can be inferred then your thoughts and actions become unpredictable in any sense. And, it is the connection between thought and action that is the key. People rarely keep their beliefs in their heads. No, they act on them. But, in a world where our actions have an impact on others, it is vitally important that we be able to predict (within reason) how people will act given what they believe. If their beliefs are inconsistent, we have no way of predicting what people will do.

Being able to infer anything means being able to believe that anything you do can be justified as well. Are you starting to see the problem here? If your beliefs are inconsistent it removes any impediment to you believing that any outrageous or harmful action could be justified. And, if you believe your action could be justified you're more likely to act that way.

So, how can we determine if our beliefs are consistent or not? Let's look at that next.

Unfortunately, there's no simple test for consistency but there are things you can do.

A good first step is to examine the set of statements you're wanting to test and make sure they are all true. Mind you, this is different from asking yourself whether you believe them or not. If the set of statements you're examining is your beliefs then, of course, you believe them! But, believing something is different from it being true.

For each belief ask yourself why you believe it. What is the evidence that your belief is based on? Is this good evidence? How do you know this is good evidence?

This can be quite difficult but it is key to determining not only whether your beliefs are consistent but also whether they are well-grounded.
What you might discover as you begin to analyze your beliefs is that some of them cannot be true if you also hold others to be true. This is the mark of inconsistency.

For example, I believe the Earth is round but I also fear sailing the ocean because I might fall off the edge of the Earth. But, believing the Earth is round precluded being able to fall off the edge (round things don't have edges to fall off of!).

So, once I find an inconsistency I have a decision to make. I can't hold both beliefs to be true at the same time. So, which do I reject?

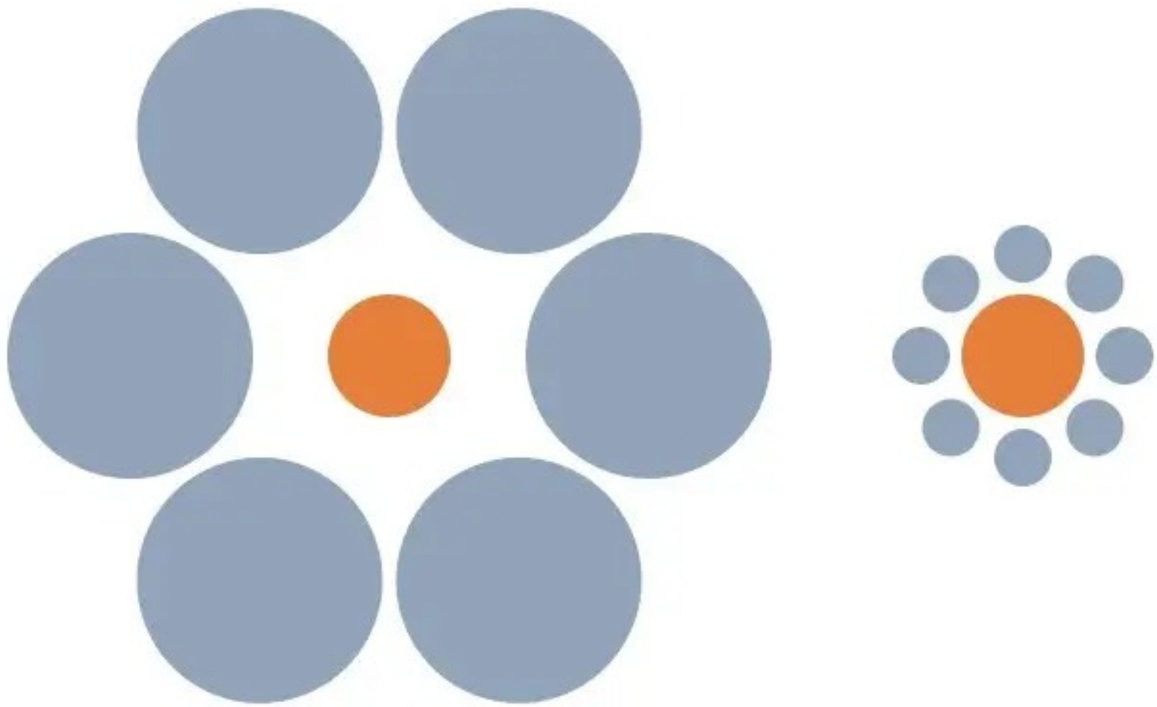
Logically, you should reject the one(s) which have the least backing by good evidence and reason. But, as the article, I link to in the Consistency lesson folder illustrates this is not always easy to do.

Test Your Consistency: Philosophical Health Check

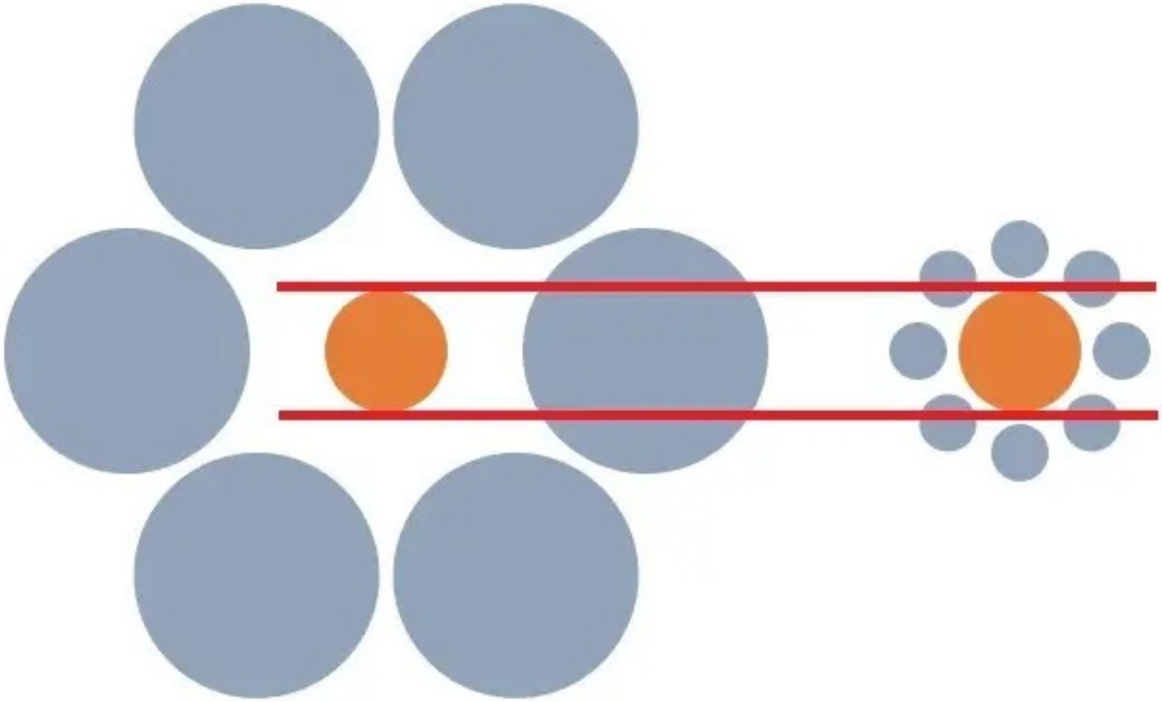
Facts? We Don't Need No Stinking Facts!

COGNITIVE BIASES

Take a look at this picture:



Which orange circle is bigger? The one of the right?
Now, look at this picture:



It clearly shows that both circles are the same size. So, now you know you've made a mistake in your perception. You know that both circles are the same size.

So, when you look back at the first picture you will see the truth about the size of the circles right? Try it.
What happened? I'll tell you what happened. You still saw the circle on the right as larger than the one on the left. You can't not see it this way. Even though you now know they are both the same size!

Think about this for a second. You perceive something. You draw a wrong conclusion from your perception. That you made a mistake is clearly shown to you but you still can't see it. This is how optical illusions work.
But, what if it's also how cognitive biases work? In fact, it is.

Cognitive biases are mistakes we make in our reasoning process when we draw wrong conclusions for a variety of reasons which are illogical. Some of these are related to our particular human psychology. Others are related to our lack of experience with logical inferences. But, all of them can be shown to be wrong and not based on reason and evidence.

The thing is, that once we learn this (which we will be doing in this section) that knowledge will not automatically allow us to correct these biases. They will still be there and still be as persuasive as ever.

What we have to do is the same thing we have to do to combat optical illusions. We have to tell ourselves that even though we are seeing things one way, the reality is much different. We have to make a conscious effort to ignore our first intuition and rely on reason and evidence instead. Even if this goes against our most basic intuitions about reality. Does that sound difficult? You bet! Critical thinking does not come naturally!

8 Common Thinking Mistakes Our Brains Make Every Day and How to Prevent Them

58 Cognitive Biases That Screw Up Everything We Do

PREDICTABLY IRRATIONAL

Here's an example of something you've probably done before. You're shopping at Amazon.com and you have $30.00 worth of merchandise in your shopping cart. When you check out your total (with shipping) comes to $37.00. You can get free shipping if you spend at least $35.00 so you look for another item to add to your cart. You find one priced at $12.00 and add it. Now your total (with free shipping) comes to $42.00. So, have you saved any money? No! You've spent more merely to get free shipping.

This is a perfect example of what economists call irrational behavior. You were trying to save money and in the process ended up spending more. The interesting thing about this example, and many others we could cite, is that they are predictable. That is, we all are susceptible to cognitive biases, but our responses to them are not random. We are, to use the title of Dan Ariely's book, Predictably Irrational: The Hidden Forces That Shape Our Decisions.

A common viewpoint students express in this class and others I teach is that everyone thinks differently. Within a narrow range of options, this is true. But, it is very definitely false in the larger scheme of things. Ariely's research shows that the kinds of mistakes we make in our reasoning process are not random and not all that variable. In fact, the entire idea of cognitive biases is based on the idea that our thinking patterns have more in common than they do differences.

There should be nothing very surprising about this since we are all wired up the same as human beings. Our brains are configured pretty much the same and the variations occupy a relatively narrow range.

So, if you're like most people you don't think you're like most people. But, you're wrong about this!

Here are some other things you probably think about yourself that you are quite likely wrong about. Don't feel bad about this because you're not alone. We're all wrong about many of these things and we're all wrong in basically the same way.

You probably think you're an above-average driver. In fact, the majority of people think this. But, it can't be the case since "above average" is a category the majority cannot fit into.

You may also think you have above-average intelligence and are above average in how you look. Again, most people think this way and most cannot be right about it.

If you're like most people you probably overestimate how well you can plan things, underestimate how much time you will need to get a given task done, and procrastinate more than you think you do.

Behavioral economists like Dan Ariely study how we are predictably irrational and how to improve our decision-making given our propensity to succumb to cognitive biases. Check out the link on cognitive biases and how to avoid them for some useful tips.

Finally, if you're like most people you easily fall prey to confirmation bias. This is one of the most important cognitive biases to understand and guard against. We all have the propensity to only look for evidence in favor of our beliefs and ignore the evidence against our beliefs.

This occurs in most areas of debate. If you follow political discussions you have seen this (and probably also fallen prey to it). You know the arguments for your belief and you think they are good arguments. You also "know" that the arguments for the other side of the debate are bad arguments and the people who hold these beliefs are ill-informed. But, this is what the confirmation bias leads us to think.

In fact, most people don't have a clear understanding of both sides of whatever issue they are debating. They don't realize that there are good arguments for the other side and that people hold these positions for good reasons. That doesn't mean that all arguments are equal and that there aren't bad arguments. But, we tend to see the bad in ideas that other people hold, not our own. This can be very dangerous, especially when this bias leads us to make bad decisions in our own lives or our social policies.

COMMON LOGICAL FALLACIES

We need to consider some common mistakes in reasoning; what we call fallacies. We should attend to these for two reasons. First, we want to be able to identify these mistakes if they occur in the philosophical arguments we will be analyzing. In fact, identifying fallacies may come in handy when considering non-philosophical arguments as well. Second, we should familiarize ourselves with fallacies to avoid committing them ourselves.

Aristotle originally identified many of these fallacies and later philosophers added them to the list. Many of these can occur in everyday argumentation but here are the more common ones to watch out for.

1. Ad Hominem: (Sometimes referred to as argument against the person)
a. attack the person (ad hominem abusive)
b. question the person's motives for making the argument (ad hominem circumstantial)
c. accuse the person of hypocrisy (tu quoque)
In these fallacies, the arguer attacks the person instead of attacking the person's argument. It is often very difficult to keep these two separate. Does the following seem like a good argument?

Bill Gilmore has argued for increased funding for the disabled. But nobody should listen to that argument. Gilmore is a slob who cheats on his wife, beats his kids, and never pays his bills on time.

In fact, it is not a good argument because instead of criticizing Gilmore's argument for funding the disabled the criticism is about Gilmore himself. In fact, it is irrelevant to his argument whether or not he cheats on his wife, etc. While these may be reprehensible things, they do not necessarily mean his argument for increased funding for the disabled is a bad argument.

2. Ad Populum: (Sometimes referred to as appeal to the people) the line of reasoning here is: "Accept the conclusion of my argument because everyone else does."
The problem with this argument is that the mere fact that everyone else believes something (or does something) is not a sufficient reason for you to accept the conclusion of your argument.

3. False Appeal to authority: While appeals to authority can be effective arguments, consider several points:

Is the source an authority on the subject at issue?
Is the source biased?
Is the accuracy of the source's observations questionable?
Is the source known to be generally unreliable?
Has the source been quoted?
Can the source's claim be settled by an appeal to expert opinion?

Is the claim highly improbable on its face?

4. Ad Ignorantiam: (Sometimes referred to as appeal to ignorance) Imagine I began my argument with the admission that I have no evidence. Then, I attempt to draw a conclusion from this lack of evidence. Sound like a good line of reasoning? Probably not. That happens in arguments that commit the Ad Ignorantiam (appeal to ignorance) fallacy.

The fallacy has the form: no one has proven this therefore we can conclude something about this. Another way to think about this is that the arguer is claiming that since no one has proven the claim false, it must be true (or vice versa, since no one has proven the claim true, it must be false). But in every case, the problem stems from the lack of evidence. In general, it is fallacious to conclude anything based on a lack of evidence. It's like saying "I've never tried beets before, but I know I won't like them." How do you know if you have no evidence?

I said this line of reasoning is fallacious in general because there are a couple of important exceptions to this fallacy. Consider this argument: Teams of researchers have been searching for life on Venus for years and have failed to find any evidence of such life. So we may conclude that there is no life on Venus.

This is, in fact, a good argument, even though it sounds like an appeal to ignorance. The difference is the "team of researchers." If qualified researchers are searching for evidence this means they are probably qualified to find it and if they fail to do so this lack of evidence itself may be important. Therefore, we can draw a conclusion from it. This doesn't guarantee that the conclusion is true but no strong argument can guarantee that.

The second exception is connected with the presumption of innocence made in our legal system. The text has good examples of that under the appeal to ignorance fallacy so I won't duplicate that material here except to mention the exception.

5. Tu Quoque: Here's an example of the fallacy:
Dr. Morrison has argued that smoking is responsible for the majority of health problems in this country and that every smoker who has even the slightest concern for his or her health should quit. Unfortunately, however, we must consign Dr. Morrison's argument to the trash bin. Only yesterday I saw none other than Dr. Morrison himself smoking a cigar.

Yes, this does make Dr. Morrison a hypocrite. But, the question is does it make his argument against smoking a bad argument? No. While it would be better if he practiced what he preached the fact that he doesn't does not mean that what he is preaching is wrong.

6. Hasty Generalization: This fallacy occurs when you attempt to draw a conclusion about a whole group based on a few members of that group. Of course, not all generalizations are hasty but many which use very few individuals to generalize are. This fallacious line of reasoning is the basis for many stereotypes.

7. Biased Statistics: Sometimes this fallacy is covered under the Hasty Generalization heading. A common mistake in statistics is using a sample that is too small to draw any conclusions from. In good statistical reasoning, the sample is random, representative, and large enough from which to draw useful conclusions.

8. Bifurcation: (Sometimes referred to as false dichotomy) Sometimes called false dichotomy is a fallacy that relates to disjunctive syllogisms. These arguments are valid but unsound because the either/or premise (the dichotomy) is false. This is a fallacy of presumption because the false either/or statement presumes that there are only two options. For example, Either I get into law school or my life is over. This would be an example of a false either/or statement. This fallacy can be committed even if the dichotomy is all that is stated in the argument.

Usually, it is obvious what conclusion should be drawn so often the argument is not completely stated. Some of the examples in the text are not completed but let's consider one that is:
Either you let me attend the Dixie Chicks concert or I'll be miserable for the rest of my life. I know you don't want me to be miserable for the rest of my life, so it follows that you'll let me attend the concert.

The problem here is with the either/or statement. Surely, there's at least one other option! Remember though that not all dichotomies are false.

9. Begging the question: occurs in two ways. First, it can occur when you leave out a questionable premise in your argument, a premise that is required for the argument to work. For example, Given that murder is immoral it follows that abortion is immoral. Now, regardless of your view on abortion, it should be obvious that there's a premise missing from this argument: Abortion is a form of murder. Without addressing this premise the argument is really not addressing the central issue. Another way of thinking about this is to say that the argument is not answering the central question involved; in other words, it's begging the question.

A second way this fallacy is committed is by arguing in a circle by having the conclusion of your argument serve as one of the premises as well. This is often referred to as circular reasoning. The example concerning Ford Motor Company in the text is a good example of this type of begging-the-question fallacy.

10. Post-Hoc Ergo Propter Hoc: A lot of causal arguments are based on the fact that if one event occurs first and a second event follows the first event must have caused the second one. This, of course, is not always true.

11. False Cause: One version of false cause is the Post Hoc Ergo Propter Hoc fallacy but false cause is a more general category. Many cases of false cause occur when correlation is confused with causation.

12. Red Herring: In red herring what usually happens is that the critic just drifts away from the original subject; hoping that the audience will forget what the original argument was! Here's an example:

Environmentalists argue that the use of pesticides on fruits and vegetables is dangerous to our health. But, fruits and vegetables contain many essential nutrients that can prevent disease and promote health. According to the FDA, one of the best sources of vitamin C is orange juice, and vegetables like broccoli contain a healthy dose of minerals such as iron. Clearly, fruits and vegetables are important to our health.
Now, what was the original argument? It was about pesticides. But, then look what happens. The critic proceeds to change the subject to talk about vitamins and minerals. This has nothing to do with whether the environmentalists are correct about the dangers of pesticides! Classic red herring fallacy.

13: Slippery slope: this is sort of a variation on false cause. The main difference is that where false cause arguments are based on one cause, one effect, slippery slope is based on one cause which leads to a series of effects. So it has the form:

If A happens, then B will happen which will lead to C which in turn will lead to D. We don't want D to happen, so we better not let A happen. The problem is that it is very unlikely that the series will occur given the occurrence of A.

14. Equivocation: In equivocation, there is a single word in the argument that is used in two distinct ways. For example, the law of gravity is a law, and laws can be repealed by the legislature. Therefore, the law of gravity can be repealed.

The problem with this argument is that it uses the word "laws" in two different senses. So, it equivocates on the word “law."

PRACTICAL APPLICATIONS

Fallacies are among the most practical parts of logic. But, still, there are useful lessons to consider which are not directly addressed by examining the fallacies. Here's a good example from Elliot Cohen's book [What Would Aristotle Do?]:

"Here's a reality check. Fill in the blanks:

If you don't win then you _______________
If you are not smart then you are ____________
If something is not true then it's ____________
If something is not good then it's ____________
If you're not happy then you are _____________
If you are not beautiful then you are __________

Hint: The answers are not (1) lose, (2) stupid, (3) false, (4) bad, (5) sad, and (6) ugly. In fact, if you gave any of these answers or their equivalents, then you were doing black or white thinking."

This, of course, is a variation on the false dichotomy fallacy. And it is a very common fallacy. Of course, in a logic textbook when you see it, the fallacious reasoning seems obvious. But consider how often we think in terms of dichotomies. We do this because sometimes dichotomies are true. But, it is important to remember that they're not always true and we often get into trouble by thinking that they are. It's easy to forget that many things are neither black nor white but gray.

Another good resource for learning this lesson is something called "fuzzy thinking." It doesn't sound too rigorous or logical but in fact, it is a different approach to logic based on the idea that most things are shades of gray. As Bart Kosko points out in his book [Fuzzy Thinking] the truth of the matter is that black and white are not the rules but the exceptions to the world of gray. In his view, everything is a matter of degree. Take for example being smart. Everybody is smart about some things and dumb about others. So the terms smart and dumb are rarely accurate in absolute terms. They are terms of degree. This is also Cohen's point about the fallacy of black or white thinking.

You might be objecting to the first example saying that in a game everyone is either a winner or a loser. But, remember that games and sports in general create this artificial dichotomy for a specific purpose. In life in general the absolute dichotomy between winning and losing rarely holds true.

Of course, there are other valuable lessons to learn from the fallacies. For example, the basis of most stereotypes and prejudices is some variation on the hasty generalization fallacy. Most superstitions are based on false cause. In fact, many con artists use false cause as a way of selling you bogus medical cures for everything from the common cold to cancer.

Politicians are masters of three fallacies in particular: argument against the person, straw man, and red herring. Being able to spot these fallacies may make us more informed voters which could have the effect of improving the political candidates themselves. Perhaps I'm being a little too optimistic but surely you can see the application of knowing these fallacies!

I'M RIGHT, YOU'RE WRONG!

As human beings, we have a unique ability to see things in different ways. Two people can look at the same thing at see differences. Or, the same person can see differences when looking at the same thing at different times. This raises several interesting philosophical questions. What are things really like independent of how we see them? Can we ever know which perspective is the correct one?

Of course, for many people, the answer to the second question is very easy. The correct perspective is theirs! We often find it difficult to imagine how anyone could see things otherwise or that there could be any validity to a perspective other than our own. This inability contributes to many of our most contentious debates on topics of politics and religion.

The capacity for empathy is, in part, the ability to take the perspective of another person. It is a very powerful skill and one that is critical to good thinking. While it doesn't come easily this skill can be learned. But, it requires being open to asking a few difficult questions.

What if my view on this topic is wrong?
What if there is another equally valid viewpoint?
What if there is information I am missing which would cause me to change my perspective?

Here are some thinking tips from the C.I.A. which can also help with this:

1. Become proficient in developing alternative points of view.
2. Do not assume that the other person will think or act like you.
3. Think backward. Instead of thinking about what might happen, put yourself into the future and try to explain how a potential situation could have occurred.
4. Imagine that the belief you are currently holding is wrong, and then develop a scenario to explain how that could be true. This helps you to see the limitations of your own beliefs.
5. Try out the other person's beliefs by actually acting out the role. This breaks you out of seeing the world through the habitual patterns of your own beliefs.
6. Play "devil's advocate" by taking the minority point of view. This helps you see how alternative assumptions make the world look different.
7. Brainstorm. A quantity of ideas leads to quality because the first ones that come to mind are those that reflect old beliefs. New ideas help you to break free of emotional blocks and social norms.
8. Interact with people of different backgrounds and beliefs.

If you look closely at this list and compare it with your everyday life you will see that you mostly don't do these things. In fact, most people mostly don't do these things. We tend to associate with people we already agree with, read material we already agree with, and watch media with views we already agree with. So, it becomes very difficult to even imagine someone thinking differently. And, next to impossible to imagine that someone could think differently for good reasons.

But, keep in mind that since everyone else thinks this way as well there are people who are listening to and watching media with which they agree but who disagree with whatever view you hold. And, they believe the same thing about your view! Breaking out of this limiting perspective is an important part of becoming a good critical thinker.

MINDSET

Interestingly, we have a great deal of control over how we see things. We can choose our perspectives. And it turns out that with regard to these choices some are better than others.

One school of ancient philosophy, called Stoicism, taught this very idea. Philosophers like the Roman Emperor Marcus Aurelius, Seneca, and Epictetus all taught that we can improve our happiness by choosing the best perspective to view both the good and bad events that occur in our lives.

A few examples:
"If you are pained by any external thing, it is not the thing that disturbs you, but your judgment about it. And, it is in your power to wipe out this judgment now." Marcus Aurelius

"People are not disturbed by things, but by the views which they take of things." Epictetus

" No man finds poverty a trouble to him, but he that thinks it so; and he that thinks it so, makes it so. He that is not content in poverty, would not be so neither in plenty; for the fault is not the thing, but in the mind." Seneca

What each of these sentiments has in common is the idea that while we may not be able to choose what happens to us we can always choose how to view it or how we feel about it.

This ancient idea has been borne out by contemporary psychological research. In particular, the research of Carol Dweck and her work on "fixed" and "growth" mindsets.

To understand this distinction think about something you've had difficulty with recently. Perhaps a class you're taking. Now, ask yourself what is the cause of your difficulty. Is it because you are just not good at the subject? You just don't have a talent for it? Or, is it because you haven't put forth the necessary effort and practice to master it? If you answered that it's because you're just no good at the subject you have a fixed mindset. On the other hand, if you think your difficulty is the result of not putting forth the necessary effort, and that if you just worked harder you could master it, you have a growth mindset.

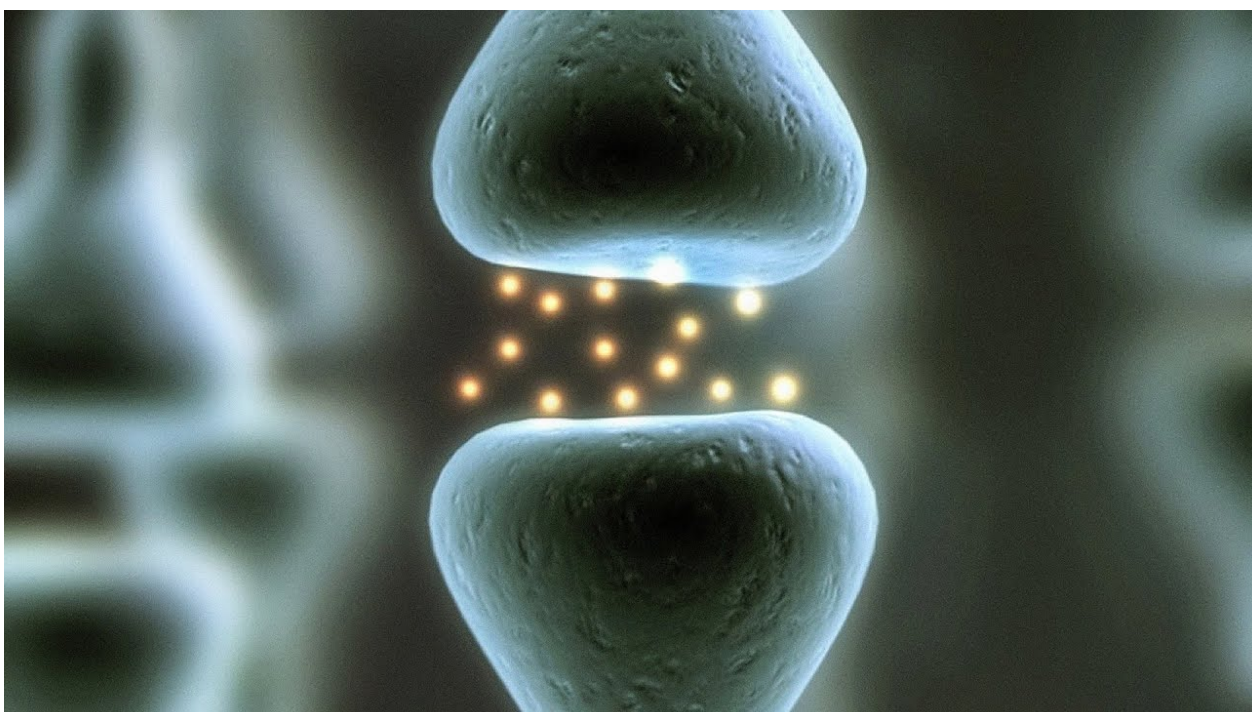
"So what?" I hear you saying. Who cares why I'm having trouble? The point is that I can't master the subject. But, the real point is that the reason you think you're having difficulty can have a large impact on the possibility of success. A fixed mindset virtually precludes success from the very beginning. If success in any given subject or activity is due to inherent ability then if you don't have the ability there's nothing you can do. You might as well go ahead and give up now!

On the other hand, if success is not simply a matter of inherent talent but a matter of hard work and deliberate practice then there is a possibility of success and that possibility depends on your action. A growth mindset provides the opportunity for success.

The kicker is that whichever attitude you choose to adopt will be correct. It's up to you. And, your chances for success (and happiness) are much higher with a growth mindset.

The golfer Jack Nicklaus once told a story about a conversation he heard in the locker room before the beginning of a golf tournament. One player was complaining about the conditions of the golf course, the wind was too strong, the bunkers too wet, the greens too fast, and so on. The player dreaded playing in these conditions. Another player was talking about the very same conditions but with a sense of anticipation. He was excited about facing the challenge of playing under such conditions. Nicklaus knew that the first player's chances of succeeding were much less than the second player.

Objectively, the challenges we all face may be the same. But, how we choose to view them is very different. It is important to remember that you have a choice in how to view things.



UNIT FOUR: PROBABILITY & PROBLEM-SOLVING

ANECDOTAL EVIDENCE

In his book Microtrends: The Small Forces Behind Tomorrow's Big Changes, Mark Penn writes about our ability to really know what is going on in the world. As Penn points out, "an average person cannot tell the difference between 4 percent unemployment and 8 percent unemployment." Given how many people we know and that we mostly get our information from these people or observe them we simply cannot tell the difference between an economic boom and a recession (which would be the difference between 4 percent and 8 percent unemployment).

Most people get their news from highly unreliable sources including various websites, chats with friends, the occasional news show, and "their own gut." That means, surprisingly, that "most people end up being wrong much of the time about what is actually going on." It's not for lack of facts that people end up wrong but lack of interest in the facts.

I remember several years ago a conversation with a friend of mine regarding the rate of divorce. I was mentioning the fact that the divorce figure most often cited (50% of marriages end in divorce) was flawed and did not accurately reflect the number of people actually divorced. She responded that it must be true and perhaps even higher because virtually everyone she knew was divorced.

The flaw in this thinking is due to our over-reliance on anecdotal evidence. We use our own personal experiences as evidence for all sorts of things when this evidence is actually very biased. With regard to the divorce example above the problem with using “everyone I know” is that everyone I know may not be representative of the population at large. Virtually everyone I know is college educated or is going to college. From that, I might conclude, wrongly, that the vast majority of Americans are college-educated. But, that is simply untrue. Using anecdotal evidence based on my personal experience has led me to a wrong conclusion.

It seems counterintuitive, doesn’t it? What could be more reliable than my own experience? But, as we’ve seen so far in this course there are quite a lot of problems with our own experience. We are susceptible to cognitive biases of all sorts and we are predictably irrational.

But, there’s another problem as well to consider. We often don’t see what we think we see and we often don’t see things that are really there. To see what I mean, watch this:

How did you do? If you'd like to see some of the other videos you can go to their website here: The Invisible Gorilla. The point of their studies is that while we all think we see everything in our field of view accurately, this simply is not the case. We are selective in our attention and in being selective we often miss seeing what's right in front of us. This is a major reason to be wary of anecdotal evidence.

This flaw, or cognitive bias, in our perception, illustrates the problem with using eyewitness accounts and personal experience as the basis for drawing conclusions. So, you've had an unexplained remission of a serious illness, you've perceived strange sights and noises that cannot be explained naturally, you've had premonitions, visions, or a near-death experience. All of these seem extremely compelling and reliable. But, the simple fact of the matter is that they are not reliable if you want to draw conclusions from them regarding what is happening in the world. The fact of the matter is that there are explanations for all of these experiences and good reasons for doubting that they are reliable indications of the external phenomena they seem to be indicating.

This is why experiments regarding the efficacy of new drugs or scientific theories regarding how the world works use peer review, double-blind protocols, and large representative samples. We need to use these devices to avoid the problems inherent in anecdotal evidence and ensure that the conclusions we are drawing are accurate. Isn't this what we really want when it comes to understanding the world? Accuracy and reliability.

PROBABILITY

As we discussed in an earlier lesson, inductive arguments are based on probability. While there are many different types of inductive arguments each has in common that the premises provide probable support for the conclusion. Given that these kinds of arguments are very common it is a good idea to have a firm grasp of the basics of probability. You may already have a working knowledge of these basics, especially if you've ever taken a course in statistics. If not, this should be very helpful and provide a useful foundation for us to build on with more interesting ideas and applications.

The relevant distinction in logic is between necessity and probability. Few things in life are really necessary in the logical sense. Logical necessity is when two things must go together and their failure to do so gives rise to a contradiction. So, for example, it is a logical necessity that a bachelor is an unmarried man. You cannot be a married bachelor. This is a contradiction. Merely thinking of the concept "married bachelor" gives rise to the contradiction.

Now, as strange as it might seem many physically impossible things are logically possible. It may seem impossible that the sun will not rise tomorrow. But, there is nothing logically contradictory about the sun not rising (yes, technically the sub doesn't rise at all since it is the Earth that is doing the moving but you get the point!). So, logically speaking the sun's rising is a probable event since it is not 100% guaranteed. Anything short of 100% is merely probably.

That being the case, it should be clear that there are degrees of probability. It makes sense to say that some events are more probable than others. Some events are less probable.

It should also be clear that there is an inherent uncertainty involved in probability. We can't know for sure that an event will happen if there is a probability less than 100% attached to it. An event can be highly likely, yet still uncertain since certainty relates to being 100% sure.

Probability affects people, places, and events, in the aggregate and doesn't provide us with as much data regarding individuals. So, if you know that there is a 1 in 6 chance that rolling a die will yield a 1 that speaks to the probability in general. It doesn't tell you that if you roll a die 6 times 1 of those will be a 6.

Understanding these basics is very important because it allows you to really grasp the information that can be provided from statistics. People often believe that statistics are unreliable and can be used to draw any conclusions. But, this is only the case if you do not have a good grasp of what probability is and what statistics can (and therefore cannot) do. Before illustrating let's cover some basic definitions. Mind you, this won't be a comprehensive lesson on statistics (you're welcome!) but should give you enough of an overview to understand the basics of statistical reasoning.

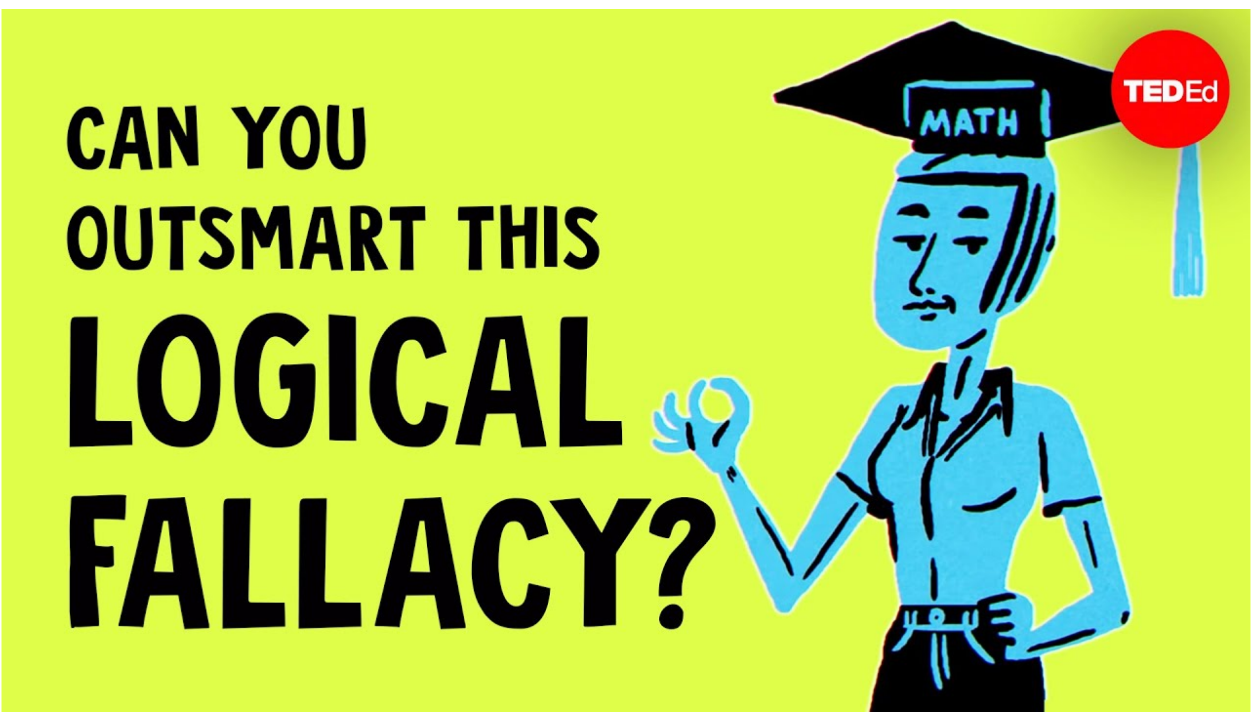
Average: As you may know, the concept of "average" has several definitions in statistics and it is important to know which is being used to understand any statistics that cite averages.

Mean: This is the arithmetical average of any given distribution. So, if someone tells you that the mean age of a population is 55 you know that what they've done is added all the ages of everyone in the population (or sample) and then divided by the number of people in that population (or sample).

Median: This represents the midpoint in a distribution. So, for example, if someone says that the median income in the United States is $35,000 a year that means half the population earns more than this and half earns less.

Mode: This represents the largest data point in a distribution. So, if the modal grade in a class is 76 that means that 76 was the grade earned by the most students in the course.

In some cases, the mean, median, and mode will be the same for a given population but this is not always the case. That is why it is crucial to know which number is being used when someone talks about the "average."



BEYOND THE BASICS

Let's start with a few scenarios which are taken from Jon Allen Paulos' excellent book titled Innumeracy: Mathematical Illiteracy and Its Consequences:

Judy is thirty-three, unmarried, and quite assertive. A magna cum laude graduate, she majored in political science in college and was deeply involved in campus social affairs, especially in anti-discrimination and anti-nuclear issues. Which statement is more probable?

a. Judy works as a bank teller.
b. Judy works as a bank teller and is active in the feminist movement.

Here's another:

You're at a party with 23 other people. What are the chances that two of these 23 people share the same birthday?
a. 1 chance in 365
b. 1 chance in 1,000
c. 1 chance in 2
d. 1 chance in 2,020

If you're like most people you have a strong intuition about the answers to these questions. And, if you're like most people your intuition is wrong. Human beings are not good at determining probabilities. This is a crucial insight to understand if you're going to gain any real grasp of statistics, their use, their abuse, and understanding how the world works.

Consider the first question. You probably think statement (b) is more probable but that cannot be the case. All things being equal more general claims are more probable than more specific claims. Even knowing what you know about Judy given the description, it is still more probable that she is simply a bank teller. One reason for this is the simple fact that there are more bank tellers than there are bank tellers who are active in the feminist movement.

The second question is tougher to see but the answer is, perhaps, equally counter-intuitive. The probability of any two people out of twenty-three having the same birthday is 1 in 2. In other words, 50%. There is a somewhat complex mathematical proof for this claim but you can also go at it another way. Think about the probability of 50%. It's not very high. Granted it's higher than the other listed probabilities but it's still pretty low. Think about it. If you were told going into this class that you had a 50% chance of passing what would you do? Probably drop the course!

You may have thought that 1 in 2 was too high because you were comparing it to the other given answer choices. You saw they were much lower and figured the answer couldn't possibly be the highest probability. This is called the framing bias. We tend to make decisions based on how the choices are framed as opposed to the inherent merits of the choices.

What these examples illustrate is that we often conclude what we believe the probability of an event to be without bothering to determine the actual probability. But, probability is not a matter of belief or guessing. There is always a method for determining the probability of an event whether that event is independent (that is the probability of the event occurring is not related to the probability of another event), dependent (where the probability of the event depends on the probability of another event), or conditionally probable (which involves the probability of one event given that another event has occurred).

MORE ON PROBABILITY

One of the most interesting implications of these insights is that events that we often regard as quite improbable, perhaps even extraordinary and supernatural turn out to be very probable. As Aristotle once said, "it is likely that unlikely things should happen." To see why let's consider some examples.

Two major misconceptions relating to probability are that a) probable events have to happen and b) improbable events can never happen. Some variation of these two misconceptions plays a role in many philosophical (and everyday) confusions.

The misconception is often due to a missing piece of information: how many chances are there for the event in question to occur? The easiest example to illustrate this relates to getting pregnant. According to WebMD the chances of getting pregnant in any given month (for a woman trying to get pregnant) is 15-25%. Some estimates of the chances of getting pregnant from any individual unprotected sexual encounter are between 3-5%.

From this, you might conclude that getting pregnant is a very rare event. But, look at the world population and you realize that it is not as rare as these statistics suggest. What's going on here? Is this a case of statistics giving us an unreliable account of how the world works? No, it is a case of missing an important piece of data. To illustrate keep in mind that 85% of couples who have unprotected sex get pregnant within a year. What is the missing piece of information? Frequency of sexual activity.

Here are a couple of other examples where the question of how many chances are there for the event in question to occur is relevant.

One obvious example is the lottery. The odds of any individual winning who plays the lottery are quite small but the odds of someone winning are much higher. This is because so many people are playing the lottery. The more individuals there are who play the lottery the more chances there are that one of them (probably not you!) will win.

Here's another example. Suppose you're playing the card game bridge. You are dealt a hand of 13 cards. The odds of getting the exact hand you were just dealt are less than 1 in 600 billion. So, from this do you conclude that this event did not happen? No. After all, you were just dealt that exact hand. The mere probability of being so low does not mean such events cannot happen. Think of it this way. Since you were dealt a hand of cards you were 100% certain that you would be dealt some hand of 13 cards. Every hand has a low chance of being drawn but you had to get one of them. It's very similar to the example of getting pregnant.

Here's a more difficult example from John Allen Paulos' book on innumeracy:

"Assume there is a test for cancer which is 98% accurate (i.e. if someone has cancer the test will be positive 98% of the time and if someone does not have cancer the test will be negative 98% of the time). Assume further that .5 percent -one out of two hundred people- actually have cancer. Now imagine that you have taken the test and your doctor soberly informs you that you've tested positive. The question is: How depressed should you be?"

Think about that for a moment. I'll give you the answer in the next lesson.

But, what about the really outlandish improbable events that happen? Here are some examples from [How to Think About Weird Things]:

"A woman finds herself thinking about an old friend she hasn't thought about for ages or seen in twenty years. Then she picks up the newspaper and is stunned to see her friend's obituary.

"A man reads his daily horoscope, which predicts that he'll meet someone who'll change his life. The next day he's introduced to the woman he eventually marries.

"A woman dreams in great detail that the house next door catches fire and burns to the ground. She wakes up in a cold sweat and writes down the dream. Three days later her neighbor's house is struck by lightning and is damaged by fire."

Could all these things happen by chance? Let's look at it.

CHANCE AND RANDOMNESS

If you've followed the thinking this far you know the answer to the question posed at the end of the last lesson. Yes, these things could be (in fact, are) the result of chance. The probabilities in any given case may be low but the number of chances for these events to occur is quite high.

Just consider the example of the prophetic dream. As Schick and Vaughn point out in How to Think About Weird Things, "a normal person has about 250 dreams per night and over 250 million people live in the United States (this figure should be higher now than when the book was written; over 300 million). So, "there must be billions of dreams dreamed every night and trillions in a year." With so many chances for dreams and real-life events to coincide isn't it quite likely that there will be many such occurrences?

When presented with any such "improbable" event you need to be sure to ask the right questions to accurately judge the probability of the event in question. Don't simply ask what are the chances of this single event occurring. That won't give you sufficient information. Also, be sure to ask how many chances there are for events like this one to occur. With regard to an example, I mentioned in a previous lesson, yes the odds that someone at a party who shares your birthday are quite low (much lower than 50%) but the odds of any two people at that party (of at least twenty-three other people) sharing a birthday are 1 in 2.

We do not sufficiently appreciate that chance and randomness play a major role in the events in our lives. We naturally seek out patterns and causal explanations and sometimes impose these on what are nothing more than random events.

Here's an example from Thomas Kida's book Don't Believe Everything You Think. "Which of the following sequences of Xs and Os seems more random (e.g. flipping a coin)?

1. XOXXXOOOOXOXXOOOXXXOX
2. XOXOXOOOXXOXOXOOXXXOX

Most people say the second is more random. However, the Xs and Os switch 70% of the time in the second case whereas they switch 50% of the time in the first case." So, the first case is the truly random case. But, look closely at this random distribution. It includes things you would not intuitively expect in a random outcome. The streak of 4 Os leaps out. How can this be random? But, the fact of the matter is that we're just not good at recognizing randomness. This is an important insight that is well explored in Leonard Mlodinow's book The Drunkard's Walk: How Randomness Rules Our Lives.

OK, so back to the cancer question:

"Assume there is a test for cancer which is 98% accurate (i.e. if someone has cancer the test will be positive 98% of the time and if someone does not have cancer the text will be negative 98% of the time). Assume further that .5 percent -one out of two hundred people- actually have cancer. Now imagine that you have taken the test and your doctor soberly informs you that you've tested positive. The question is: How depressed should you be?"

You're probably thinking you should be quite depressed. After all, the test is 98% accurate. But, you should actually be "cautiously optimistic." The reason why illustrates the kind of thinking you need to do to really understand probability.

"Imagine that 10,000 tests for cancer are administered. Of these, how many are positive? On average 50 of these 10,000 (.5 of 10,000) will have cancer, and so, since 98 percent of them will test positive, we will have 49 positive tests. Of the 9,950 cancerless people, 2 percent of them will test positive, for a total of 199 positive tests (.02 x 9,950=199). Thus of the total 248 positive tests (49+199=248), most (199) are false positives, and so the conditional probability of having cancer given that one tests positive is only 49/248 or about 20 percent!"

PROBLEM-SOLVING PART 1

Like critical thinking, problem-solving is one of those skills that are in high demand. Many employers rate it as one of the most important traits they look for in prospective employees. It is also usually listed as an important learning outcome in many of the classes you take including math classes, psychology classes, and even this class! But, it is often not taught at all or very well.

One reason why this is the case is because of a confusion relating to what kinds of problems are being solved. It is often assumed that by practicing any kind of problem for which you don't have an answer you will learn the skills appropriate to solve any problem. But, this is simply not true. Since there are different types of problems learning how to solve one type won't necessarily help you build the skills you need to solve the other type. So, let's begin by distinguishing two types of problems: puzzles and mysteries.

In his book Curious: The Desire to Know and Why Your Future Depends on It, Ian Leslie defines these two types of problems very well: puzzles and mysteries.

"Puzzles have definite answers. Puzzles are orderly; they have a beginning and an end. Once the missing information is found, it's not a puzzle anymore." Most of the problems you encounter in your college math courses are, in reality, puzzles. They have definite answers and are orderly. Also, the information you need to solve them is right there in the question! And, if you need extra help you can just turn back to the chapter in the text which discusses how to solve these particular types of problems.

Many "logic puzzle" questions are (obviously) also just puzzles. While they seem more complex, in reality, they are the same as your math problems. They have definite answers and are orderly. Here are a couple of examples:

You have eight billiard balls. One of them is "defective," meaning that it weighs more than the others. How do you tell, using a balance, which ball is defective in two weighings?

You have five jars of pills. All the pills in one jar only are "contaminated." The only way to tell which pills are contaminated is by weight. A regular pill weighs 10 grams; a contaminated pill is 9 grams. You are given a scale and allowed to make just one measurement with it. How do you tell which jar is contaminated?

Another important characteristic of puzzles is that you can quite often Google the answer for them! Go ahead try and figure these two out and if you have trouble just Google the answer.

Now, here's the question. Are these kinds of problems "real?" In other words, are these the kinds of problems you're likely to run into in your everyday life? Are they the kinds of problems you will run into at work? I doubt it. Another question then: Is learning how to solve these kinds of problems a good preparation for learning to face real-world problems? Again, I doubt it.

So, the second type of problem Leslie defines is mysteries. "Mysteries are murkier; less neat. They pose questions that can't be answered definitively because the answers often depend on a highly complex and interrelated set of factors, both known and unknown." In other words, the exact kind of problems you're likely to run into in real life and at work.

PROBLEM-SOLVING PART 2

So, how do you solve a mystery? Well, if you've understood the distinction between puzzles and mysteries you can probably guess what I'm about to say. There's no easy method that works for all mysteries. Their very ambiguity and lack of clarity ensure that this is so. But, there are things you do can do to improve your ability to solve these kinds of problems. Here are a few steps to take.

1. Practice solving mysteries. Like any other skill the more you practice solving these kinds of problems the better you will become at solving them. And, it should be easy to find such problems as they crop up in everyday life all of the time. Here are some possible examples:

[A] How can I achieve a better balance between all my responsibilities including work, school, and family?
[B] How can I save more money for retirement or other future expenses than I'm currently saving?
[C] How can I find a career that will allow me to earn enough money to support myself and my family and also allow me to grow and build my skills and talents?

I know what you're thinking. Sure, these are real-world problems and I've even faced some of them but how can I build the skills I need to solve these problems before taking these problems on? It does no good to practice these very difficult problems.

So, to prepare for solving these kinds of problems here are some other potentially helpful steps.

2. Learn as much as you can about as many topics as you can. Solving mysteries often involves drawing on knowledge from a variety of sources. It is quite likely that many of the things you're now learning (even those you think are irrelevant) may turn out to be helpful at some point in solving a problem. So, take advantage of the opportunity to really learn about psychology, biology, history, mathematics, and all the other subjects you're learning now. That time will pay off.

3. Make connections. A lot of problems are solved by finding interesting, creative connections between seemingly unrelated topics. A good example here is Steve Jobs' application of his knowledge of calligraphy to the Apple operating system. Another is the psychiatrist Jeffrey Schwartz's development of a treatment for OCD (obsessive-compulsive disorder) by combining Buddhism and Austrian economics.

4. Read books about mysteries, problem-solving, and even mystery novels. Sherlock Holmes is a great place to start. Reading Conan Doyle's stories of the great detective reveals some basic principles of problem-solving which can apply to everyday life. Maria Konnikova's book Mastermind: How to Think Like Sherlock is another good resource.

5. Solve simpler mysteries first. Rather than taking on the more complex type of real-world problems mentioned above, begin with simpler, but still real, problems. Nevermind solving your work-life balance problem right off. Start with solving this problem: How can I keep from losing my phone every time I get ready to go somewhere?

6. Adopt a problem-solving mindset. Be observant of your surroundings and attuned to what you can do to improve how you live, work, or do simple things. Thomas Edison exemplifies this mindset very well. He often presented his lab assistants with the following challenge. He would hand them some ordinary item (like an iron or a fountain pen) and say "There's a better way. Find it." Adopt that mentality in your life. Whatever you're doing ask whether there is a better way: easier, more efficient, more effective.

7. Learn about design thinking. Lastly, you may want to learn about design thinking. I've made a few resources available that provide a brief introduction to the basic ideas of design. The important feature is to solve problems with the end-user in mind. When you're trying to solve a problem be sure you understand for whom you are solving it and what their needs are. That will allow you to focus on the best solutions given what they're facing.

Design Thinking From the d School

UNIT FIVE: PROPOSITIONAL LOGIC

PRACTICAL APPLICATIONS

Here's an example of a logic puzzle: You have four cards (each with a letter on one side and a number on the other side). The cards you have are an A, D, 3, and 6. Now the question is which cards do you need to turn over to test the following hypothesis: cards with vowels on one side have even numbers on the other side?

OK, before you answer that question here's another question given by Malcolm Gladwell in his book [The Tipping Point]: Suppose four people are drinking in a bar. One is drinking Coke. One is sixteen. One is drinking beer and one is twenty-five. Given the rule that no one under twenty-one is allowed to drink beer, which of those people's IDs do we have to check to make sure the law is being observed?

How is this relevant to chapter six you ask? What it illustrates is something Gladwell calls the problem of context. We tend to think and problem solve differently in different contexts. While the majority of people get the first question wrong, the majority of people get the right answer for the second question. The interesting thing is that in terms of form the questions are identical!

Part of the difficulty of learning formal logic is the lack of context. When we look at truth tables (and when we look at proofs in the next unit) we are looking at thinking in the abstract separate from any given context. The point is to analyze the principles of thinking. But, it makes things difficult because as humans we are used to thinking in context.

The presumption of logic is that there is value in looking at thinking in the abstract precisely because it is free of context. If we only train ourselves to think in contexts then we are apt to miss the fact that a set of rules which works in one context will also work in another context. It's this lesson of the transference of rules that logic is really teaching. So, as you go through this unit and the next be thinking of specific contexts in which these principles of thinking apply but also remember that the point of logic is to show that these rules are universal and apply the same way in many different contexts. Not only is this an important lesson in logic, but in learning as well.

Another interesting point to make as we enter into a completely symbolic language is that you are already well acquainted with using such languages. Consider text messaging. Here's a very recent example of a fairly abstract, complex, rule-governed symbolic language. To outsiders, the code looks about as meaningless as propositional logic looks to you now! There are strange terms such as JK, LOL, CUL8R, ILBL8, as well as even stranger ones like ;-) and :-0. But, the origin of this language has its roots in practical circumstances and so does symbolic logic. In fact, many of the same reasons apply including more efficient communication. So, keep this in mind as we begin our journey into propositional logic.

Oh, the answer to the first question: You need to turn over the A and the 3. I assume you know which people you need to ID at the bar.

PROPOSITIONAL LOGIC

The next logical system we'll investigate is called propositional logic. As we'll see, it is more formal than the previous system and is also more abstract. You might wonder what the point of such a logical system is. Propositional logic developed, in part, out of the desire to advance logic in the same way that mathematics had advanced in the late 19th century as a result of a variety of work being done in geometry and mathematics.

At the time there was a great interest in the foundations of mathematics and a work by Bertrand Russell and Alfred North Whitehead called *Principia Mathematica* purported to show that all of mathematics could be reduced to a few basic principles in logic (very much like the principles in propositional logic). So the question became, Can we do the same with ordinary language? If we could, that might allow philosophers and scientists to come up with a more rigorous and accurate language to describe the world around us. With the advent of quantum physics in the early 20th century, you can certainly understand how a very pure and accurate formal language might help formulate scientific theories.

OK, that's all fine and well for scientists but what benefits are there for us to understand and use such a formal symbolic language. Well, for a start, anyone interested in computer programming will benefit greatly inasmuch as the basis of all computer languages is logic. In essence, every computer language is a formal logical language. But, everyone can benefit from the thought processes involved in learning a logical system such as propositional logic. Learning such a system requires abstract thinking skills, the ability to apply rules to various situations in order to solve problems, recognize patterns, and think in general terms.

Each of these skills is useful no matter what your interest or occupation is. Think of formal logic, in part, as an exercise regiment for your mind. The analogy I use is with working out. If you work out perhaps you lift weights or work on a stair master. Why do you do these exercises? Is it to become better at lifting things or climbing stairs? Not really. The main benefit is something else; good cardiovascular health for instance. So, the real benefit of learning and using propositional logic is not to go around using P's and Q's and translating arguments into symbolic form to prove their validity (though this is useful too!). The real benefit is the type of thinking you need to use in order to master this system. With that in mind, the next two units will develop the basic principles of propositional logic. Let's begin with the symbols involved and how to translate words into these symbols.

There are a few important differences between categorical logic and propositional logic.

First, propositional logic contains five types of statements as opposed to the four statement types in categorical logic.

Secondly, propositional logic uses variables differently. In categorical logic, letters (such as S and P) stood for terms (nouns and noun phrases). In propositional logic, letters will stand for simple declarative statements.

So "Today is Saturday" can be represented by the letter "S," "It is raining" can be represented by "R," and "We will have logic class" can be represented by the letter "L." In order to symbolize compound statements (statements that contain simple statements as one of their parts), we need logical operators. Four of these operators are called connectives since they connect two propositions. So, we have:

. conjunction (symbolized by a dot)
v disjunction (symbolized by a lower case v)
⊃ implication (symbolized by a sideways horseshoe)
≡ biconditional (symbolized by a triple bar)

~ negation (symbolized by the tilde) This symbol is different since it doesn't connect two propositions but always precedes what is being negated. We use this symbol to translate negative statements (using the words "not" or "it is not the case").

So if I wanted to symbolize the sentence: "Today is Saturday and we will not have logic class" I would write S . ~L

Notice I used the tilde to symbolize "we will NOT have logic class." Anytime there is a negative word in a statement you MUST use the tilde. There are a few potentially confusing cases of translation to watch for.

Consider this statement: Today is not Saturday and it is not raining.

We'd translate this as: ~S . ~R

But what about this?: It is not both Saturday and raining.

Not both must be translated as follows with parentheses: ~(S . R)

It is very important to recognize that these two propositions are NOT equivalent and cannot be interchanged when translating from words to symbols. Before we address equivalence consider another potentially confusing example:

Either today is not Saturday or it is not raining.

We'd translate this as: ~S v ~R

But what about: It is neither Saturday nor raining. Neither/nor (which is identical to not either) must also be translated with parentheses as: ~(S v R)

Again these two propositions are NOT equivalent. So which ones are equivalent? Well, oddly enough the equivalence works like this:

~S . ~R :: ~(S v R)
~S v ~R :: ~(S . R)

(The four dots :: means "is equivalent to").

This will become clear in the next unit when we explain DeMorgan's rule.

Another translation case to be aware of involves the use of conditional statements.

We'll start with the easy case:

If today is Saturday then we will not have logic class.
S ⊃ ~L

Now, how about this one? Today is Saturday only if we will not have logic class.

It's actually the same so: S ⊃ ~L

But what about: Today is Saturday if we will not have logic class.

This one's different and MUST be translated as: ~L ⊃ S

That is when the word "if" comes in the middle of a statement to translate it the letters MUST be reversed. This is the ONLY case where this reverse translation must be done.

We have only one more concept to address and this is the Well-Formed Formula (WFF). What this refers to is the fact that there are correct and incorrect ways to write propositions in logic.

This is similar to mathematics. For example, if I said to write the following math statement: add two and four, you wouldn't write it like this: +2 4. You'd put the addition symbol in between the numbers: 2+4. So the following kinds of propositions are not allowed:

A ⊃ B v X (you need parentheses here to illustrate the main operator)

(X v Y)(A ≡ B) (you need a symbol between the parentheses here)

$\sim\sim\sim A \supset B$ (you cannot have three negations though two are OK).

TRUTH FUNCTIONS

The heart of propositional logic is the truth function. Essentially, this is a common-sense notion. In order to determine the truth of a compound proposition, you need to know the truth of its constituent elementary propositions. In other words, the truth of the compound proposition is a function of its parts. To illustrate consider this example: I like apples and I like oranges.

We can symbolize this as: A . O

Now let's consider the truth of this statement. Suppose I really do like apples and I really do like oranges. so: A is true and O is true. So the conjunction of A . O is: true.

Suppose, on the other hand, I really like apples but don't really like oranges. So A is true but O is false. The conjunction of A . O is: false. Another possibility is that I don't like apples and I do like oranges: A is false and O is true. In this case, the conjunction of A . O is also false. And finally, if I don't like either one; A is false and O is false. The conjunction is: false. This can be summed up as a truth table:

p	q	p . q
T	T	T
T	F	F
F	T	F
F	F	F

Next, let's consider the disjunction using the example: Either I like apples or I like oranges symbolized as: A v O.

Now, what might the truth function look like for this proposition? It should match with our idea about how disjunctions work in ordinary language. That is if one or both of the parts are true the whole proposition is true. On the other hand, if both A and O are false the disjunction is false. This can be summed up in the following truth table:

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

For the conditional statement we'll use the example in the text on. This is just an example! "If you get an A on the final then you'll get an A in the course." We can symbolize this as: A ⊃ C

So let's start with the easy cases. If you did get an A on the final and you did get an A in the course then you'd say that A ⊃ C is true right? And if you got an A on the final but did not get an A in the course you'd say A ⊃ C is false. So far the truth table would look like this:

p	q	p ⊃ q
T	T	T
T	F	F

OK the other two options involve you not getting an A on the final; that is A is false. That means there are two logical options. You'll either get an A in the course or you won't. The question is which of these would it make sense to call false? Actually neither one. So the whole truth table looks like this:

p	q	p ⊃ q

T	T	T
T	F	F
F	T	T
F	F	T

The strange case here seems to be the one where both parts of the conditional are false but the conditional statement is true. Well, here's a better example to illustrate how this can work:

Suppose today is Monday and I say: "If today is Saturday then tomorrow is Sunday." Now "today is Saturday" would be false and "tomorrow is Sunday" would also be false. But the whole statement would be true!

The last connective is the biconditional and I don't have an example in words to illustrate the whole truth function but I do have a clever way to understand and remember it. To me the symbol for the biconditional looks like an equals sign, just one extra line. So how does the equals sign work? If both sides are the same it's a true statement. For example: 3+2=1+4. Both sides are five so it's a true statement. On the other hand, 4=7 is false for obvious reasons. The biconditional statement works the same. As long as both sides have the same truth value (whether true or false) the statement is true; otherwise, it's false. So here's the truth table:

p	q	p ≡ q
T	T	T
T	F	F
F	T	F
F	F	T

Now, the question is what can we do with this information? Well, the rest of this unit will examine this. We will be using these truth functions to determine the truth value of individual statements when we know the truth values of all the parts in it, some of the parts, and none of the parts. We'll also be determining the validity of arguments using these truth functions.

TRUTH TABLES FOR PROPOSITIONS

Now, we'll consider how to deal with propositions where all the variables are undetermined. Needless to say we will not be able to determine the one truth value for all of these propositions but what we can do is determine every possible truth value by constructing what's called a truth table. This sounds more complicated than it really is. Let's start with a simple example:

(A v ~B) ⊃ B

You always begin constructing a truth table by determining the number of lines it will contain which can be determined by the following equation: L=2n. This should read 2 "raised to the Nth power" as explained in the text. So, for a truth table containing two different letters (as the one above) there will be four lines; if a proposition contained three different letters the number of lines would be 8 (2 x 2 x 2).

Here's what the truth table for the proposition above would look like.

First the letters get defined

(A	v	~	B	⊃	B
T			T		T
T			F		F
F			T		T
F			F		F

Next, you fill in the values for the operators. It's a little like solving a math equation. We will begin inside the parentheses and work our way out to the main operator starting with the negation of B:

(A	v	~	B	⊃	B
T		F	T		T
T		T	F		F

F		F	T		T
F		T	F		F

Next, we figure the disjunction operator inside the parentheses using the truth values under A and the ~ symbol:

(A	v	~	B	⊃	B
T	T	F	T		T
T	T	T	F		F
F	F	F	T		T
F	T	T	F		F

Finally, we will compute the main operator using the truth values under the disjunction symbol and the letter B outside the parentheses:

(A	v	~	B	⊃	B
T	T	F	T	T	T
T	T	T	F	F	F
F	F	F	T	T	T
F	T	T	F	F	F

So, what can we use this for?

The first thing we'll use this information for is to classify propositions. Every proposition falls into one of three possible categories:

it will always be true (tautology)
it will always be false (self contradictory)
it will sometimes be true and sometimes be false (contingent)

We determine what kind of proposition we have by constructing a truth table and then looking at the line under the main operator. If that line is all "T" then we have a tautology; if it's all "F" we have a self-contradictory proposition. Finally, if it's a mix of "T's" and "F's" it's contingent.

The second thing we'll use the truth table information for is to compare propositions to determine whether they are logically equivalent, contradictory, consistent, or inconsistent. We encountered the concepts of logically equivalent and contradictory propositions in Unit Two: Rules of Reason. The concept here is identical though our way of determining these classifications will be different.

When you construct a truth table for two propositions in order to compare them construct the truth table as if there were only one proposition. That is, you define the letters in each proposition in exactly the same way even if the order is different in each proposition.

The two propositions being compared are $K \supset L$ and $\sim L \supset \sim K$. Notice that even though the letters are in a different order in each proposition they are defined the same: K is 2 true and 2 false and L is 1 true 1 false alternating.

After finishing the truth table we're interested in comparing the lines under the main operators of each proposition. If they are identical the propositions are logically equivalent. If they are exactly the opposite they are contradictory.

However, not all propositions will fit one of these two categories. In this case, look to see whether any line of the main operator for both propositions is true. If so, the two are consistent. If not, (and you have already ruled out there being equivalent or contradictory) then the propositions are inconsistent.

TRUTH TABLES FOR ARGUMENTS

Now that you're familiar with the concept of truth tables the next application should be easy. As we will see, we can use truth tables to determine the validity of arguments. The concept of using truth tables to determine validity is actually based on an idea that was first raised in chapter one. There are no valid arguments where the premises are true and the conclusion is false. You may want to review the rationale behind this which is in the text. There Hurley says that "the idea that any deductive argument having actually true premises and a false conclusion is invalid may be the most important point in all of deductive logic."

We can use this idea, together with the truth table of an argument, to determine its validity. How? Simple. Consider a simple example such as this one:

J ⊃ E / ~J // ~E.

We place a slash "/" between each premise and a double slash "//" between the last premise and the conclusion. If we do a truth table for this argument we can see that there is one case (the third line) where the premises are true and the conclusion is false. So this argument is invalid.

J	⊃	E	/	~	J	//	~	E
T	T	T		F	T		F	T
T	F	F		F	T		T	F
F	**T**	T		**T**	F		**F**	T
F	T	F		T	F		T	F

There only has to be one line on the truth table that contains true premises and a false conclusion to make the argument invalid. On the other hand, if there are no such cases then the argument is valid. That's all there is to doing a truth table for an argument!

There is one small problem with the truth table method that can be illustrated with the following argument example:

A ⊃ (B v C) / C ⊃ (D . E) / ~B // A ⊃ ~E

Can you see the problem? Well, if we did a truth table for this example it would take 32 lines!

That seems like a lot of work and you might be wondering if there's a shorter way to determine validity. In fact, there is. Consider this.

When we construct the truth table for this argument what will we be looking for? One example where all the premises are true and the conclusion is false. So, why don't we assume there's such a line and work backwards to try to reconstruct it. If we succeed in this that will prove that one line on the 32 line truth table contains all true premises and a false conclusion thus demonstrating the argument is invalid. On the other hand, if we do not succeed in reconstructing the line then that will show there is no such line and the argument is valid.

This is called the indirect truth table method and we'll address this in the next lesson.

INDIRECT TRUTH TABLES

In the last section, we used truth tables to determine the validity of arguments. However, as we saw, there was a flaw in this method when arguments contain so many letters that doing a truth table becomes extremely cumbersome. So, we can shortcut the process by constructing an indirect truth table. In essence, we'll be using this technique to reconstruct one line on the truth table of an argument but in reverse.

To illustrate the basic idea remember that we are beginning the indirect truth table for arguments by assuming the argument is invalid; that is we will assume that all the premises are true and the conclusion is false. Using the example I raised last time we would place a T under the main operator of each premise and an F under the main operator of the conclusion:

```
A ⊃ (B v C) / C ⊃ (D . E) / ~B // A ⊃ ~E
  T             T           T       F
```

Now we work backward to fill in the value of all the letters in the truth table. If we can do this without any contradictions we will have demonstrated that the argument is invalid. If we run into a problem in our reconstruction this shows that the argument is valid.

The basic idea of how to work backward can be illustrated as follows. Suppose we have a conjunction:

A . B and we know the conjunction is true. Can this tell us what the truth value of A and B are? Of course.

They both have to be true.

OK suppose we have a disjunction: X v Y and we know that the disjunction is true and Y is false. Can you figure out what X is? It should be true.

How about if we have a conditional statement: A ⊃ X and we know the conditional statement is true. Knowing this doesn't give us enough information to tell what A and X are but suppose we know X is false.

Now, what does A have to be? False since this is the only truth value that would make the conditional true.

OK now let's use this technique to reconstruct the indirect truth table for the above example.

If the negation of B in the third premise is true then B must be false. So, we'll put an F under that B and every other B as well (since the same letters have the same truth values).

```
A ⊃ (B v C) / C ⊃ (D . E) / ~B // A ⊃ ~E
T  F                T              TF       F
```

We can also determine the truth value of A and E from the conclusion. Since the conditional statement is false the letter A must be true and the negation of E is false which makes E true. So we fill in those truth values.

```
A ⊃ (B v C) / C ⊃ (D . E) / ~B // A ⊃ ~E
T T     F           T      T       TF     T F FT
```

Notice I placed a T under the A in the first premise and a T under the E in the second premise. Now let's turn to the first premise. We know the conditional statement is true and we know A is true and B is true. Can we use this information to determine the truth value of the disjunction and the letter C?

As of now, we can't figure out the letter C it could be either true or false. However, knowing that A is true and the conditional is true tells us that the disjunction must be true.

This is the only way to make the conditional statement true because if the disjunction were false the whole conditional statement would be false and we're trying to make our original assumption work out.

```
A ⊃ (B v C) / C ⊃ (D . E) / ~B // A ⊃ ~E
T T    F T          T     T       TF     T F FT
```

Now, we can use this information to tell us that C must be true.

```
A ⊃ (B v C) / C ⊃ (D . E) / ~B // A ⊃ ~E
T T   F T T      T T     T       TF     T F FT
```

Finally, we need to figure out the remaining variable in the second premise: D. We can use the same principle we used to figure out premise one. Again, we have a conditional statement that's true and we know the first part (C) is true. So, again the second part (the conjunction of D and E) must be true which makes D true.

```
A ⊃ (B v C) / C ⊃ (D . E) / ~B // A ⊃ ~E
T T   F T T      T T   T T T    TF     T F FT
```

So, that's it. We've reconstructed the whole line which shows that the argument is invalid.

To illustrate how the indirect truth table method determines that an argument is valid consider a simple example.

B ⊃ C / ~C / ~B

Of course, we could do a direct truth table but let's see how the shortcut method would work on this one. We begin, as always in this method, by assuming the argument is invalid:

B ⊃ C	~C	~B
T	T	F

Now we fill in the remaining values. Since the negation of C is true C must be false and since the negation of B is false B must be true:

B ⊃ C	~C	~B
T T F	TF	FT

Now, notice we have a problem in premise one. Our assumption was that the premise was true but with B being true and C being false this can't work. It's a contradiction. This is what tells us that the argument is valid.

The other application of indirect truth tables is to determine consistency. To illustrate why we might want to use the indirect truth table method here consider an example:

F = (A . ~P) / A ⊃ (P . S) / S ⊃ ~F / A . ~F

This is not an argument just four statements. And we want to know whether they're consistent or not. How can we tell? Well, if we did a regular truth table we'd just look for one line where all the statements are true. But, with four different letters that truth table is going to take 16 lines.

Instead, we can use the indirect method by assuming the statements are all consistent (all true) and working backward. This is exactly the same as the indirect method for arguments. The only difference is the assumption you begin with; all true statements. So you place a T under the main operator of each of the statements and proceed just like above.

UNIT SIX: PROOFS IN PROPOSITIONAL LOGIC

ARGUMENT FORMS

The principles of natural deduction provide a more elegant way of demonstrating validity than the truth table method and when mastered, are also much easier. This logical method also illustrates the use of formal rules applied to problem-solving which is really the practical application of this logical technique. Still, it can be a difficult technique to master and it is important to work slowly with one set of rules at a time and practice proving arguments valid. The first step to being able to solve proofs is to understand the rules, their validity, and their use.

As we discussed in Unit Two: Rules of Reason, there are several argument forms that are always valid. It will be important to understand these forms and remember them as they are the most commonly used rules of inference in propositional logic. The forms and their symbolic representations are:

modus ponens: MP

$p \supset q$
p
----
q

modus tollens: MT

$p \supset q$
$\sim q$

$\sim p$

hypothetical syllogism: HS

$p \supset q$
$q \supset r$

$p \supset r$

disjunctive syllogism: DS

$p \vee q$
$\sim p$
----

q
constructive dilemma: CD

$(p \supset q) \,.\, (r \supset s)$
$p \vee q$

$r \vee s$

Remember, we used these examples in Unit Two:

Modus Ponens:

If today is Tuesday then we will have logic class.
Today is Tuesday...THEREFORE?

Modus Tollens:

If you can beat H.H. Gregg's prices then pigs can fly.
Pigs cannot fly...THEREFORE?

Disjunctive Syllogism:

Either we will go to the beach or we will have logic class.
We will NOT be going to the beach...THEREFORE?

Hypothetical Syllogism:

If there is life on Mars then we will find evidence of that life.
If we find evidence of that life then we are not alone.
THEREFORE...

Constructive Dilemma:

If it is sunny then we will go to the beach AND if it is raining then we will go to the museum.
It will EITHER be sunny OR raining
THEREFORE...
We will EITHER go to the beach OR the museum.

The challenging part will be seeing these forms in the variations that occur within the proofs. A few hints about this might help:

In argument forms where the letters have no negations (MP, HS, CD) the arguments that use these forms can contain negations. Remember that the lower case letters can stand for any proposition, including a negated proposition. So for instance, this would be a valid MP:

~A ⊃ B
~A

B

So would this:

(A . B) ⊃ ~(X v ~Y)
A . B

~(X v ~Y)

And notice that with DS it doesn't matter which side of the disjunction is negated the conclusion will always be the other side.

A v B
~B

A

Also, this would be a valid DS:

~A v ~B
A

~B

The important thing is that they are opposites (A, ~A). The same would hold true for MT. As long as the two propositions are opposites the MT will work. Of course, the one being negated must ALWAYS be the consequent just as in MP the one being affirmed must ALWAYS be the antecedent. So this would be a valid MT:

~A ⊃ ~B
B

A

One final note: the order of the premises in these argument forms doesn't matter so be sure you can identify the form even when the premises are backward:

~A
A v B

B

RULES OF IMPLICATION PART 1

We now turn to the application of valid argument forms as rules of inference to prove more complicated arguments valid. Notice, that in this chapter we begin to write arguments differently; numbering the premises and placing the conclusion to the side of the last premise. For example,

1. ~C ⊃ (A ⊃ C)
2. ~C /~A

The point is to show that the conclusion (~A) can be derived from the premises.

To do this we use the first four rules of inference (MP, MT, DS, HS) to derive subsequent steps from the original premises until we reach the conclusion. A few points to remember. We can use the rules as many times as necessary and we can use any line on a proof (including ones we derive from the original premises) as many times as necessary.

At each step in the proof we're trying to see whether the lines we have (or derive) fit the form of the premises of any of the rules. If so, we use the rule to derive the conclusion as the next step in the proof.

So, we begin the proof above by seeing whether the two premises look like the premises of any of the rules of inference. They seem to look like modus ponens. From this rule we can derive line 3:

3. A ⊃ C 1, 2 MP

Notice that we supply the justification for this step next to the proposition. We have to do this for each step in the proof. From here we can derive line four using the rule of modus tollens since lines 2, 3 fit this form. So we derive line 4:

4. ~A 2, 3 MT

Thus solving the proof.

1. $\sim A \supset [A \vee (T \supset R)]$
2. $\sim R \supset [R \vee (A \supset R)]$
3. $(T \vee D) \supset \sim R$
4. $T \vee D$ $\quad\quad\quad\quad$ / D

RULES OF IMPLICATION PART 2

We'll now add four rules of inference to our proving rules. One of these rules should be familiar from the lesson on argument forms; the constructive dilemma:

(p ⊃ q) . (r ⊃ s)
p v q
----
q v s

This is sort of like two modus ponens done at the same time. It is very important to note that in order to use this rule the signs must be exactly as described in the rule (this is true of all rules in fact). Particularly important is that in order to use the rule of constructive dilemma you MUST have a conjunction of two conditional statements. As we'll see, if you don't have such a line in your proof you can create it using the new rule of **conjunction**.

This new rule allows you to take any two lines in your proof and combine them providing you do so using the conjunction symbol. This is commonly used to set up a CD. For example:

1. A ⊃ B
2. X ⊃ Y
3. A v X /B v Y

To solve this proof we can combine lines 1 and 2 to create the first premise of the CD.

4. (A ⊃ B) . (X ⊃ Y) 1, 2 conj.

Notice when we do this we must use parentheses to make a well-formed formula. From here we can use line 4 along with line 3 to generate the conclusion with the rule CD.

5. B v Y 3, 4 CD

Another helpful rule is called **simplification**. This can ONLY be applied to conjunctions but it is a very helpful rule. As long as the conjunction is the main operator on a line you can use simplification.

However, if the conjunction is not the main operator the proposition CANNOT be simplified. These propositions, for example, could not be simplified:

~(A . B)

A ⊃ (X . ~Y)

As we'll see in the next lesson, it is not only possible to simplify the left side of any conjunction but also the right side.

The final rule in this section might seem a little strange. It's called **addition** and looks like this:

p
----
p v q

What it asserts is that you can take any proposition in a proof and add ANYTHING to it (even if it's not already present in the proof) as long as you use the disjunction symbol. This is valid because of the truth function for disjunction which asserts that in order for a disjunction to be true only one part of the disjunction has to be true.

Here's an example of how this rule can be used:

1. (A v X) ⊃ (B . R)
2. A / B v D

So, we'll begin by adding an X to line two in order to do a MP with line one.

3. A v X 2 add

Next we'll do the MP:

4. B . R 1, 3 MP

Now how do we get the conclusion? Well, line four is a conjunction so we can simplify to the letter B and simply add the D.

5. B 4 simp.
6. B v D 5 add

Notice, we're using the new rules along with the original four rules.

1. (D v E) ⊃ (G . H)
2. G ⊃ ~D
3. D . F / M
4. D 3 simp.
5. D v E 4 add
6. G . H 1,5 MP
7. G 6 simp.

RULES OF REPLACEMENT PART 1

The rules we'll be looking at and using in this section are different from the rules of implication. They are called rules of replacement which means that anywhere in a proof they allow you to replace one proposition with its equivalent proposition.

Why would you ever want to do that? Well, in order to solve the proof it is sometimes necessary to change a proposition in order to apply such rules as MP or DS.

Let's consider the most important rule in this group and illustrate some possible uses of it.

We first introduced **DeMorgan's rule** in Unit 5 when we first looked at formal propositional logic.

This rule asserts that
~(p v q) :: ~p . ~q and also that
~(p . q) :: ~p v ~q.

The symbol :: means "is equivalent to."

To illustrate the rationale behind this rule it might help to remind yourself of the examples we used to explain this equivalence:

"You canNOT have BOTH a hamburger and a hot dog" means the same as
"You EITHER canNOT have a hamburger OR you canNOT have a hot dog."

"Today is NEITHER Saturday NOR Sunday" means the same as
"Today is NOT Saturday AND today is NOT Sunday."

Consider the following example to illustrate why we might need such rules:

1. ~(A v B)
2. A v (X . Y) / X v ~Z

To begin this proof we'll change premise 1 using DM.

3. ~A . ~B 1 DM

Now, we can simplify line three and use it with line two with the rule DS.

4. ~A 3 simp.
5. X . Y 2, 4 DS

Now what? Well all we need to do is simplify line five and add the ~Z.

6. X 5 simp.
7. X v ~Z 6 add

So, this is what we need rules of replacement for. Notice that all the rules of replacement ONLY work with conjunctions and disjunctions. These rules CANNOT be used with conditional statements or biconditional statements.

Some of them are very similar to rules in mathematics. For example:

2 + 3 equals 3 + 2. This is called commutativity in math and we have a similar rule in logic called **commutativity**:

p v q :: q v p
p . q :: q . p

Another math-like rule is **associativity**. In math 2 + (3 + 5) = (2 + 3) + 5.

In other words, if the signs are the same parentheses can be moved from front to back or back to front. So, in logic:

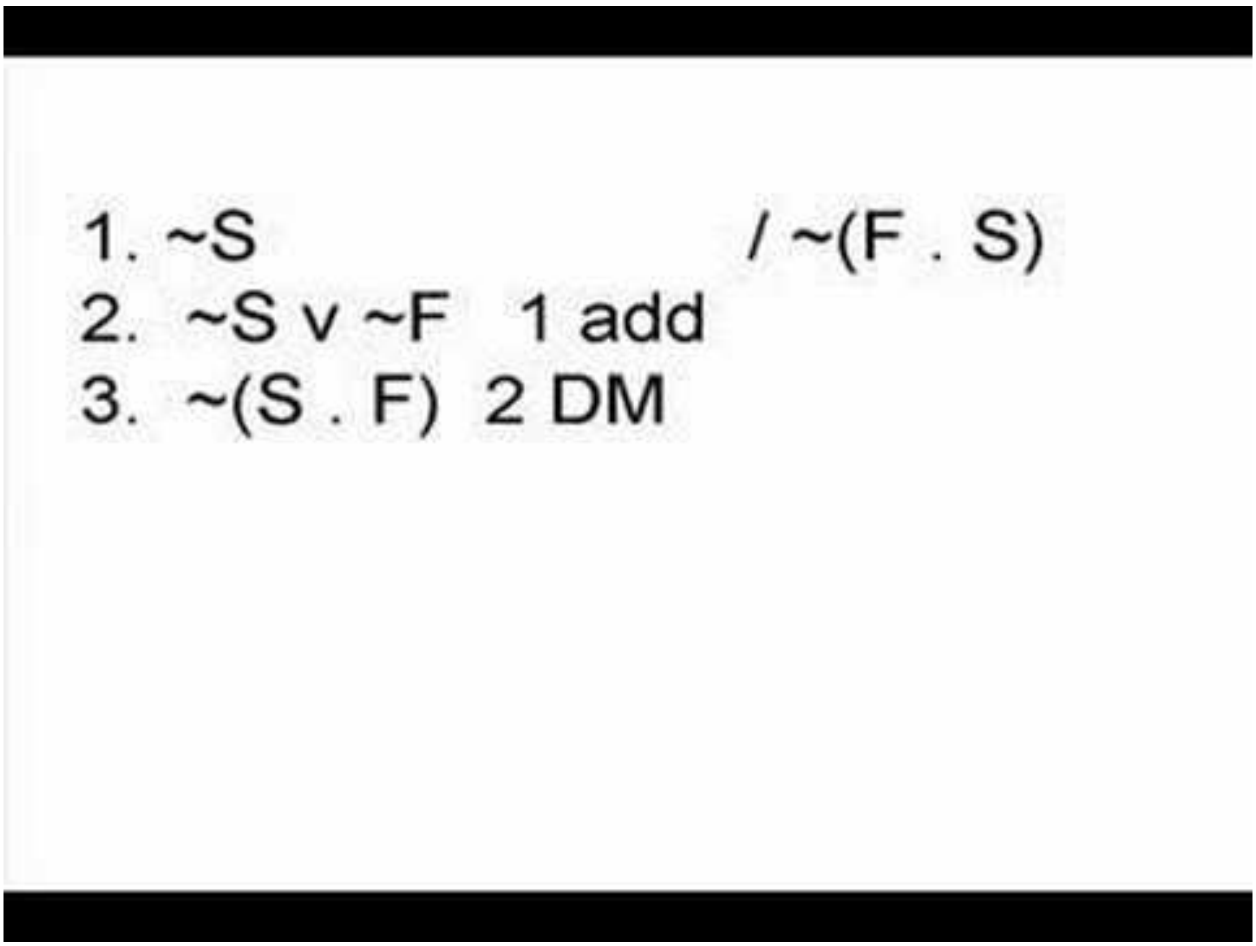
p v (q v r) :: (p v q) v r
p . (q . r) :: (p . q) . r

The final rule we'll look at in this section is the rule of distribution. (We're not going to be using the rule of double negation). Distribution is also similar to the mathematical rule but not exactly.

The rule of **distribution** is:

p . (q v r) :: (p . q) v (p . r)
p v (q . r) :: (p v q) . (p v r)

The second version of this is particularly helpful since it allows you to turn a disjunction into a conjunction which can then be simplified.



RULES OF REPLACEMENT PART 2

Our final set of rules allows us to make important changes to conditional statements and biconditional statements. One of the things you'll notice about these rules is that they allow us to make many changes and determine the validity of arguments starting with fewer premises. Given so many rules and so many options it is important to devise a strategy before trying various rules. Here's a useful tip:

If inspection of the premises does not reveal how the conclusion should be derived, consider using the rules of replacement to "deconstruct" the conclusion.

I'll illustrate this with a few examples but first the final rules. One of the most important and commonly used rules in 7.4 is called **material implication**:

p ⊃ q :: ~p v q

This rule allows us to turn any conditional statement into a disjunction or any disjunction into a conditional statement. We can do this as long as the first part of the proposition is negated so:

~A ⊃ B can be turned into: A v B. To see the sense behind this equivalence consider the following example using words:

"IF you don't leave me alone THEN I'll punch you in the nose" means the same as
"EITHER you leave me alone OR I'll punch you in the nose."

The next rule is called **transposition**:

p ⊃ q :: ~q ⊃ ~p

This is essentially commutativity for conditional statements but you must remember to negate BOTH parts of the conditional statement as you reverse the variables.

Here's an example to make sense out of this rule:

"IF that animal is a cat THEN that animal is a mammal" means the same as
"IF that animal is NOT a mammal THEN that animal is NOT a cat."

Next, **Exportation**: (p . q) ⊃ r :: p ⊃ (q ⊃ r)

Which explains this equivalence:
"If today is Saturday AND it is sunny THEN we will play golf" means the same as
"If today is Saturday THEN IF it is sunny THEN we will play golf."

The important thing here (as will all these rules) is that the signs must be exactly like the rule in order for it to work. In other words a proposition like:
$A \supset (X \,.\, Y)$ cannot be changed by exportation.

Neither could: $(A \supset B) \,.\, C$

Nor this: $(A \supset B) \supset C$.

This rule is useful to set up MP or MT or HS.

There's only one rule that addresses the biconditional statement: **Material Equivalence**:

$p \equiv q :: (p \supset q) \,.\, (q \supset p)$
$p \equiv q :: (p \,.\, q) \vee (\sim p \,.\, \sim q)$

As you can see there are two propositions that are equivalent to the biconditional statement. The first of these expresses the meaning in words for the "if and only if" statement. The second one expresses the truth-functional meaning of the biconditional. Remember that from chapter 6, the biconditional is true if both p and q are true or if both p and q are false. It is usually pretty easy to tell when this rule should be used.

This brings us to the last rule: **tautology**. It's not often used but when needed can be very helpful. It asserts that any proposition is equivalent to a disjunction of itself or a conjunction of itself:

$p :: p \,.\, p$
$p :: p \vee p$

The second is the more useful. Here's an example of this rule in action.

1. A ⊃ B
2. X ⊃ B
3. ~(~A . ~X) / B

So where to begin? Well, we can turn premise three into something by using DM.

4. A v X 3 DM

Notice the two negations cancel out and we simply drop them. Now, we have two conditional statements and a disjunction. This suggests setting up a CD.

5. (A ⊃ B) . (X ⊃ B) 1, 2 conj.
6. B v B 4, 5 CD

And finally the rule of tautology gives us the conclusion:

7. B 6 taut.

For an example of applying strategy 15 consider this argument:

1. ~R v P
2. R v ~P / R ≡ P

OK, we know that the conclusion has to come from the rule of material equivalence.

The question is which version:

(R ⊃ P) . (P ⊃ R) or (R . P) v (~R . ~P)

Well, we have two disjunctions in premises 1 and 2 and we now have the ability to turn them into conditional statements which we can then conjunct. So:

3. R ⊃ P 1 imp.
4. ~R ⊃ ~P 2 imp.

Premise 3 now looks like half of what we need but premise 4 looks nothing like the other half. But, we can use the rule of transposition to change this:

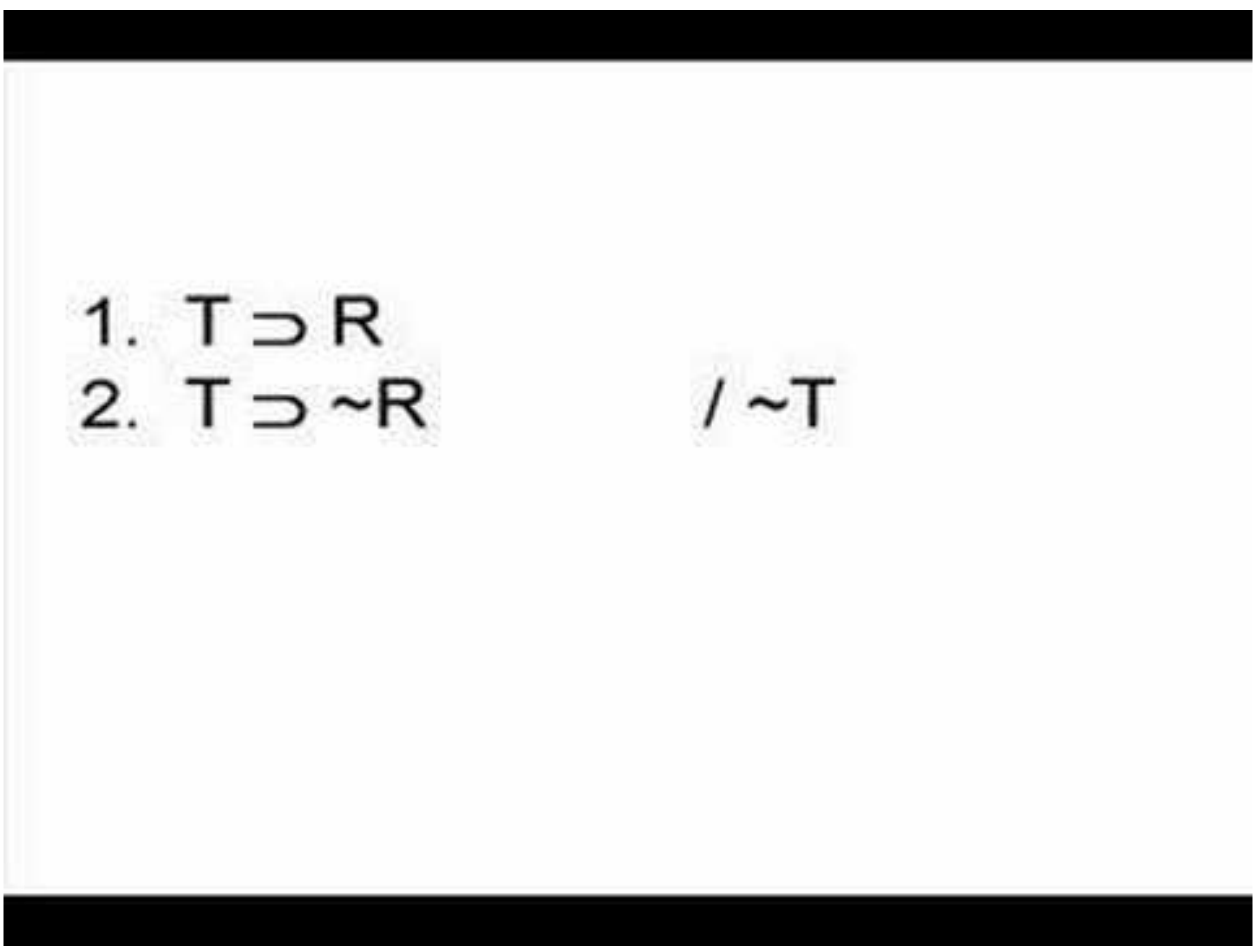
5. P ⊃ R 4 trans.

Now, we conjunct them:

6. (R ⊃ P) . (P ⊃ R) 3, 5 conj.

And finally:

7. R ≡ P 6 equiv.



PRACTICAL APPLICATIONS

These examples come from a book by Elliot Cohen titled [Caution: Faulty Thinking Can Be Harmful to Your Happiness](). These fallacies illustrate the practical application of several rules we've discussed in this unit. In particular, the rule of transposition and the fallacy of confusing affirming the consequent with MP, and confusing denying the antecedent with MT. Here's how Cohen explains them.

The Vice Versa fallacy (this illustrates the misapplication of transposition).

Look for example at the following two statements:

If my wife and I have a good marriage then my wife and I do not cheat on each other.

If my wife and I do not cheat on each other then my wife and I have a good marriage.

Thinking these two statements mean the same thing is called the Vice Versa fallacy. Hopefully, you can see how this confusion would cause problems! The truth of statement number 1 does not imply the truth of statement number 2. It is not an overstatement to say that many couples end up in counseling (or worse divorced) because they fail to see this.

Remember the rule of transposition says that $A \supset B :: {\sim}B \supset {\sim}A$. So for the example above the truth of statement number 1 would imply the truth of statement 3: If my wife and I cheat on each other then my wife and I do not have a good marriage.

Fallacy of Deriving "if" from "then" (this illustrates the flaw in thinking that the argument form of affirming the consequent is valid).

As Cohen puts it "this fallacy arises when you conclude that because the THEN-clause of an "if…then…" statement is true, then the IF-clause must also be true."

For example, "If Sam is sterile then he has not impregnated his wife." Supposing that the consequent is true (he has not impregnated his wife) does it necessarily follow that Sam is sterile? No! Think of how Sam would feel if we drew that conclusion.

Fallacy of Deriving "then not" from "if not" (this illustrates the flaw in thinking that the argument form of denying the antecedent is valid).

The example Cohen uses here is: If the traffic light is red then I have to stop. So, does it necessarily follow that if the traffic light is NOT red then I do NOT have to stop? No, there may be other reasons you should stop. Think about if a pedestrian is crossing the road or if a biker suddenly runs in front of you.

Now, you may be saying that these are obvious examples but the point is that we use these logical rules (and sometimes misuse them) every day. The more we think in terms of logic being useful the easier it is to learn. T

Now, see if you can detect the fallacies committed by the following examples:

My car keeps stalling out, and that's just what would happen if the fuel pump was bad. So it must be a bad fuel pump.

Mother to her son: "If you had a fever then you would be sick. But I took your temperature twice and you don't have a fever. So, you're really not sick. Get your tail out of bed and get ready for school you little con artist."

See if you can find other practical examples of the rules of inference in action. Thinking in terms of real examples will make it easier to learn and apply the rules.

UNIT SEVEN: FINAL PRACTICAL TOOLS

FINAL PRACTICAL TOOLS

I've referred to some of the ideas in this section previously in the course. But, they all warrant a much more detailed treatment and I think it is best that the authors speak for themselves. So, the remaining lessons in this section contain videos from several important thinkers whose ideas can help improve your thinking.

Tasha Eurich Insight

Julia Galef The Scout Mindset: Why Some People See Things Clearly and Others Don't

Michael Syed Black Box Thinking: Why Most People Never Learn From Their Mistakes

Adam Grant Think Again: The Power of Knowing What You Don't Know

Dan Ariely Predictably Irrational: The Hidden Forces That Shape Our Decisions

Daniel Kahneman Thinking Fast and Slow

William Irvine The Stoic Challenge: A Philosopher's Guide to Becoming Tougher, Calmer, and More Resilient

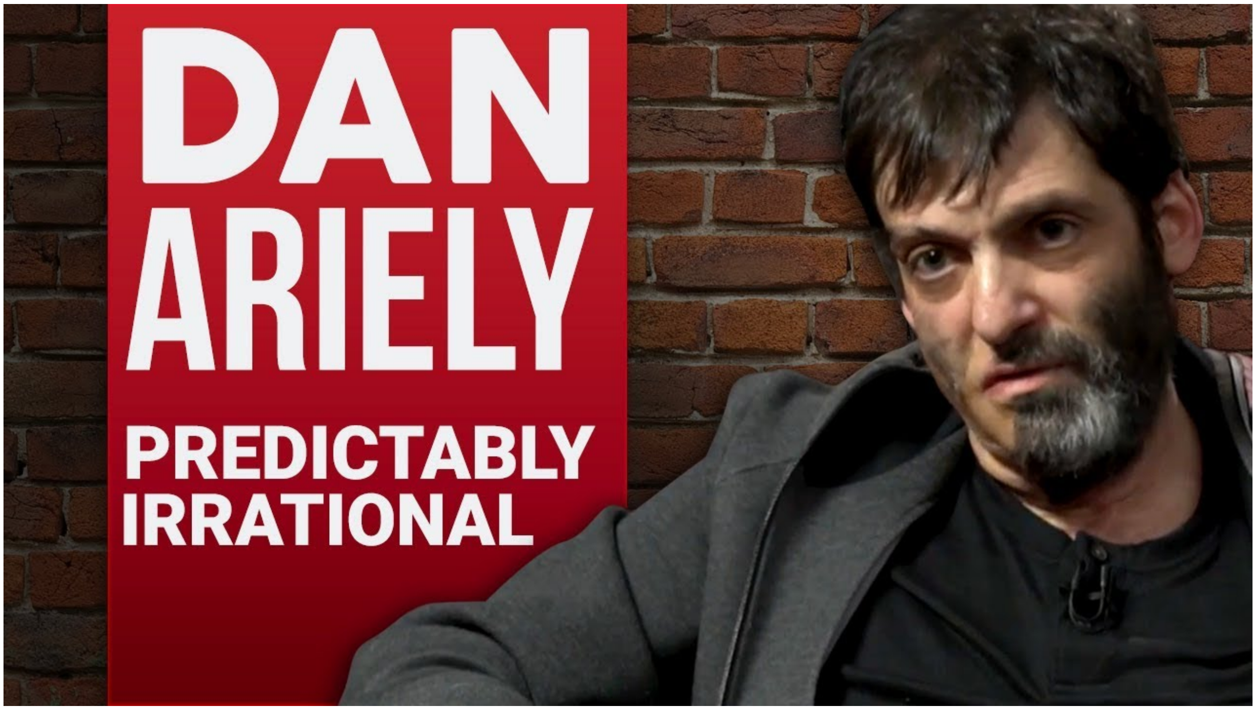
Tasha Eurich: Insight

Julia Galef: The Scout Mindset

Michael Syed: Black Box Thinking

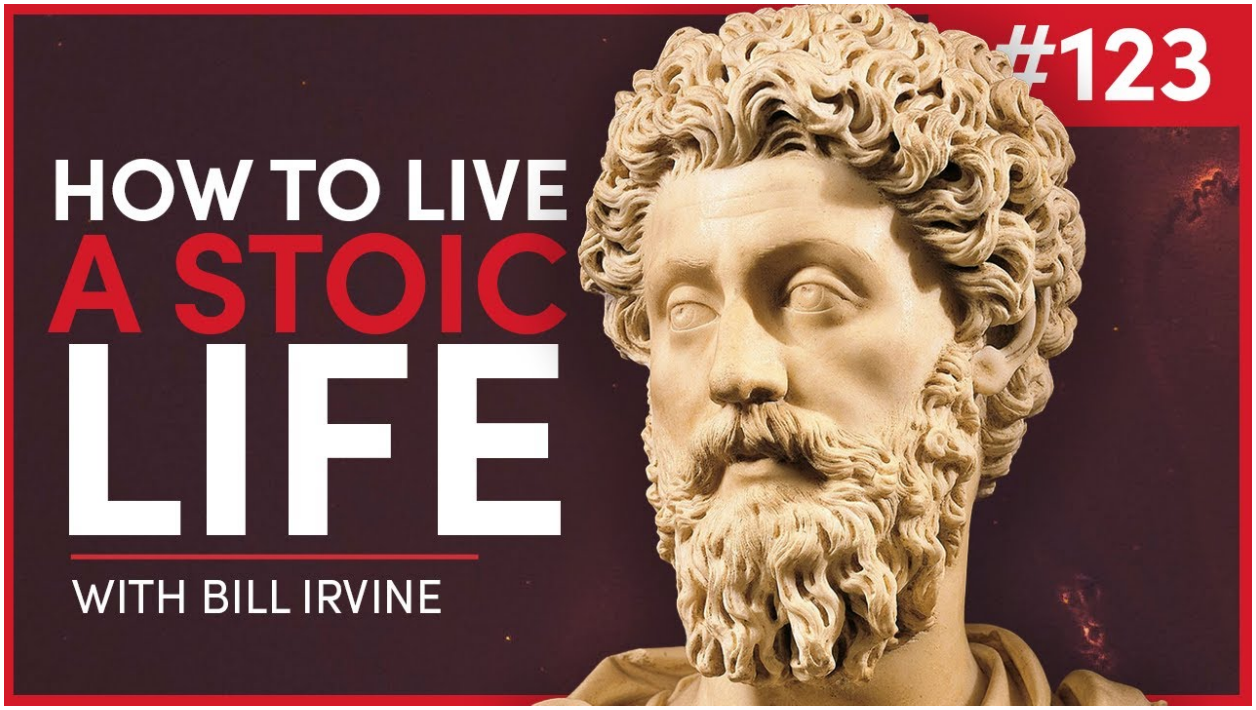
Adam Grant: Think Again

Dan Ariely: Predictablyy Irrational



Daniel Kahneman Thinking Fast and Slow

William Irvine: How to Live a Stoic Life



Julia Bogart Raising Critical Thinkers

FINAL THOUGHTS

FINAL THOUGHTS

So, there you have it! You've completed the "Improve Your Thinking" course. Remember that this course has been an introduction, not an ending. Look upon it as your invitation to continue your logical development. Thinking is not an activity that ends once you complete a class or graduate. Ceasing to think amounts to ceasing to live and as John Dewey pointed out "close-mindedness means premature intellectual old age."

The feeling many people have after being exposed to logic is the feeling of being exposed to something they'll never use. I hope throughout the course I have addressed this issue to your satisfaction. But, there is a problem. I've only addressed it from a conscious, cognitive standpoint. But, to really use logic you have to make it a part of you, and as I've said before you have to develop a logical sense. That is, you have to take the external lessons of logic and internalize them. This is a process only you can complete. I can show you the mechanics but you have to make them part of your everyday reasoning process and use them.

Of course, this does not mean that in the course of a conversation you start writing out truth tables or translate what others are saying into propositional logic and use the rules of inference to prove the argument valid. What it means is that you develop an ear for good thinking and an ear for recognizing bad thinking. Just like musicians can train their ears to hear chord progressions and poets can train their ears to hear certain meters and rhythms you can train yourself to hear the logical structure of arguments.

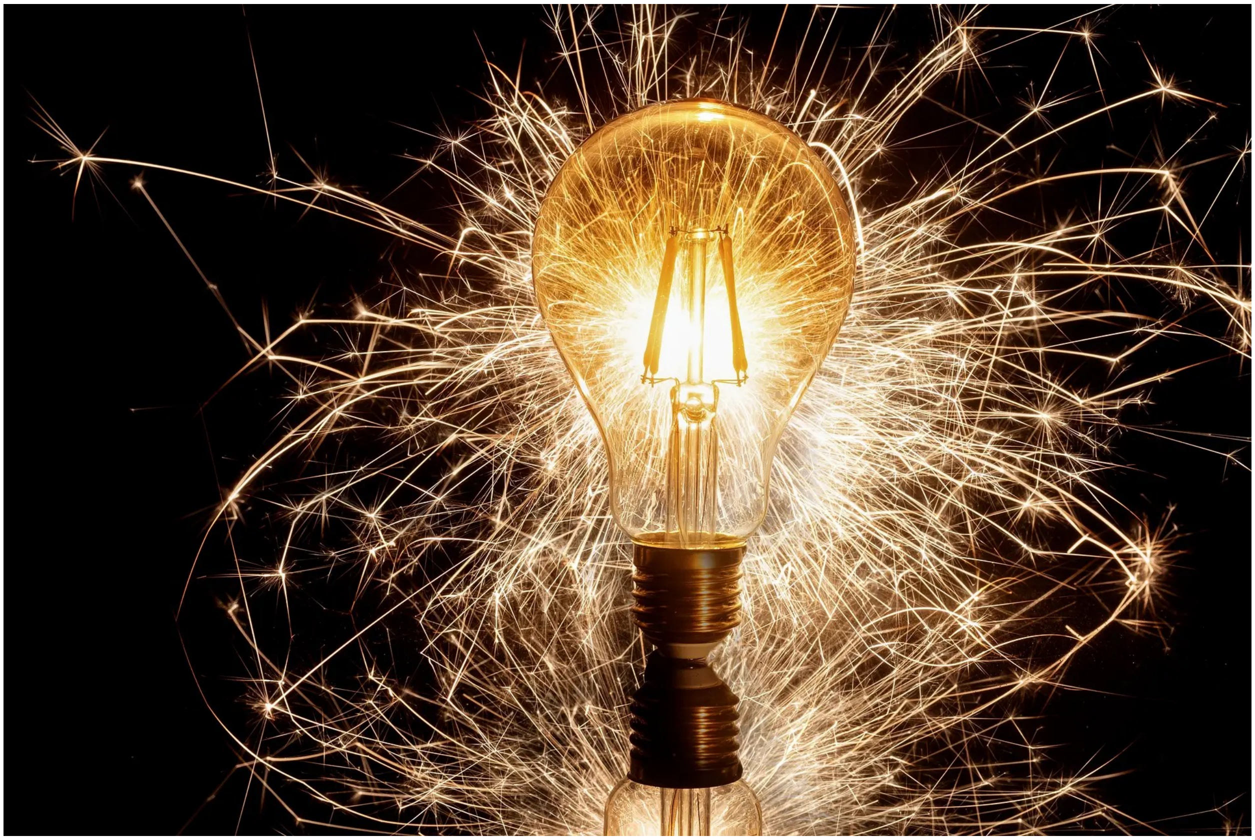
And the benefits can be enormous. Just consider the examples I have shown you throughout the semester and think about examples from your own experience. There are so many cases in life where you will have to call on good thinking and problem-solving skills but how do you go about practicing these skills in order to prepare for applying them.

Lately, many people have been practicing these skills by doing sudoku. These number puzzles are very popular but look at them in terms of the mental skills they are using. They are abstract formal reasoning skills. The very same skills we examine and practice in logic. But while sudoku puzzles challenge us in our use of these skills only logic forces us to look at the reasoning process with an eye toward understanding it and improving it. This is the true value of logic.

Each person will use their thinking skills in different ways throughout their life so the applications will be different. That's why we often don't teach logic with an eye toward application. There are simply too many to account for! The idea behind teaching logic the way we do is that if we focus on the mechanics each individual student will find their own applications.

I know this can be difficult which is why I've tried to give you examples of other people's application of logic. But, you will really only begin to see the value of logic when you find your own examples. I want to encourage you to continue to do this even after you leave the course. Feel free to e-mail me with examples that will help others learn the lessons of how to apply logic to everyday life.

Kevin J. Browne
http://kevinjbrowne.com



RECOMMENDED READING: THINKING

Michael Shermer	Why People Believe Weird Things
Theodore Schick	How to Think About Weird Things
D.Q. McInerny	Being Logical
Anthony Weston	Creativity for Critical Thinkers
Jamie Whyte	Crimes Against Logic
Malcolm Gladwell	Blink
Julia Galef	The Scout Mindset
Adam Grant	Think Again
Ori Brafman	Sway
Daniel Levitin	A Field Guide to Lies
Rolf Dobelli	The Art of Thinking Clearly
Carol Tavris	Mistakes Were Made
Maria Konnikova	Mastermind
Bart Kosko	Fuzzy Thinking
Vincent Ruggiero	Beyond Feelings
Gary Kirby	Thinking
Steven Levitt	Think Like a Freak
Shankar Vedantam	Useful Delusions
Lewis Wolpert	Six Impossible Things Before Breakfast
Ronald Gross	Socrates' Way
Steven Pinker	Rationality
Daniel Kahneman	Thinking Fast and Slow
	Noise
Dan Ariely	Predictably Irrational
	The Upside of Irrationality

RECOMMENDED READING: HABITS & DECISIONS

Charles Duhigg	The Power of Habit
James Clear	Atomic Habits
BJ Fogg	Tiny Habits
Stephen Covey	The 7 Habits of Highly Effective People
Gretchen Rubin	Better Than Before
Warren Berger	A More Beautiful Question
Zachary Shore	Blunder
Bill Burnett	Designing Your Life
Thomas Sowell	Knowledge and Decisions
Dan Pink	Drive
John Kay	Obliquity
Steven Johnson	Farsighted
Brian Christian	Algorithms to Live By
Gary Klein	Sources of Power
Jonah Lehrer	How We Decide
Cass Sunstein	Wiser
Barry Schwartz	The Paradox of Choice
Dan Ariely	Amazing Decisions
Jamie Holmes	Nonsense
Richard Thaler	Nudge
Chip & Dan Heath	Decisive
	Switch

RECOMMENDED READING: CREATIVITY

Mary-Elaine Jacobsen	The Gifted Adult
Ken Robinson	Out of Our Minds
Tony Wagner	Creating Innovators
Bruce Nussbaum	Creative Intelligence
Tom KelleyCreative	Confidence
Amy Herman	Visual Intelligence
Ed Catmull	Creativity, Inc.
Elizabeth Gilbert	Big Magic
Jonah Lehrer	Imagine
Adam Grant	Originals
Matthew Syed	Rebel Ideas
Daniel Nettle	Strong Imagination
James Austin	Chase, Chance, and Creativity
Steven Johnson	Where Good Ideas Come From
Edward de Bono	Creativity Workout
Austin Kleon	Show Your Work!
Daniel Boorstin	The Creators
David Lynch	Catching the Big Fish
Michael Michalko	Thinkertoys
	Creative Thinkering